Lecture 7

Capacitance

Roadmap

In the last lecture we showed how we can represent a voltage waveform that changes in time another way – as a sum of tones (sinewaves). This representation contains the same information, just stored in a different way. These two representations are called transforms of each other.

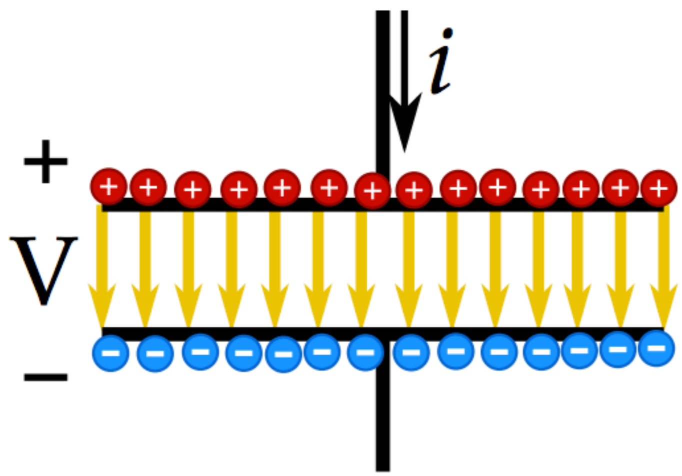
This lecture will show you why this alternative representation is so useful. If you can represent an input as a sum of sinewaves, then it is very easy to compute the output of a circuit with resistors and capacitors. To show how this is done, we will introduce the concept of **impedance**, which is a generalization of resistance. Using impedance we can then estimate how the gain of circuit can change with frequency, creating **filters**, and how the response of these filters are graphed in a **Bode plot**.

CAPACITORS

Capacitors

- Rather than relating i and V
  - Q = CV
  - Q in Coulombs
  - V in Volts
  - C in Farads

- Charge neutral
  - +Q on one lead
  - -Q on the other lead



$$Q = CV$$

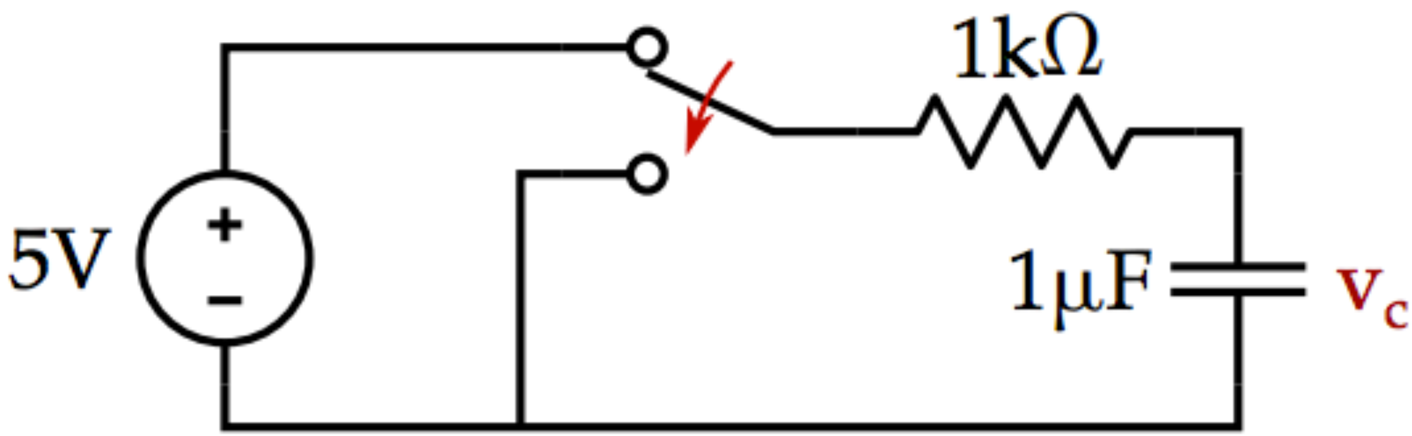
iV for a Capacitor

- How do we relate I and V though?
  - Q=CV
  - I = dQ/dt
  - dQ/dt = d(CV)/dt

- I = C dV/dt

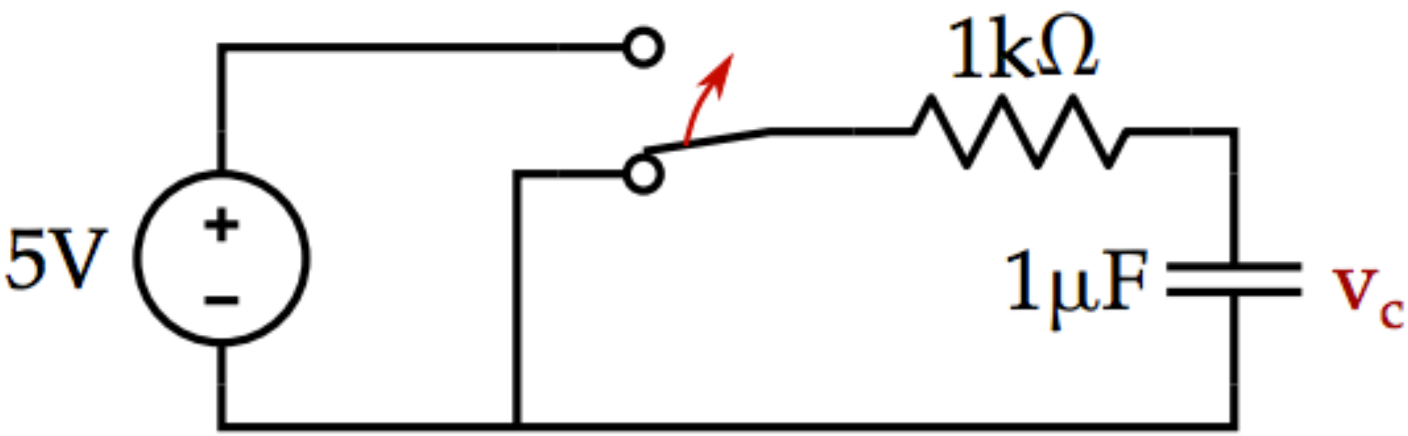
- Current = Rate of Voltage Change * Capacitance
  - If current is small, voltage changes a little bit
  - If current is large, voltage changes a lot

Capacitor Energy (Charging) – RC Circuit



- Assume Vc=0 at t=0
- At t=0, switch flips up
- Plot Vc vs t

Capacitor Energy (Discharging) – RC Circuit

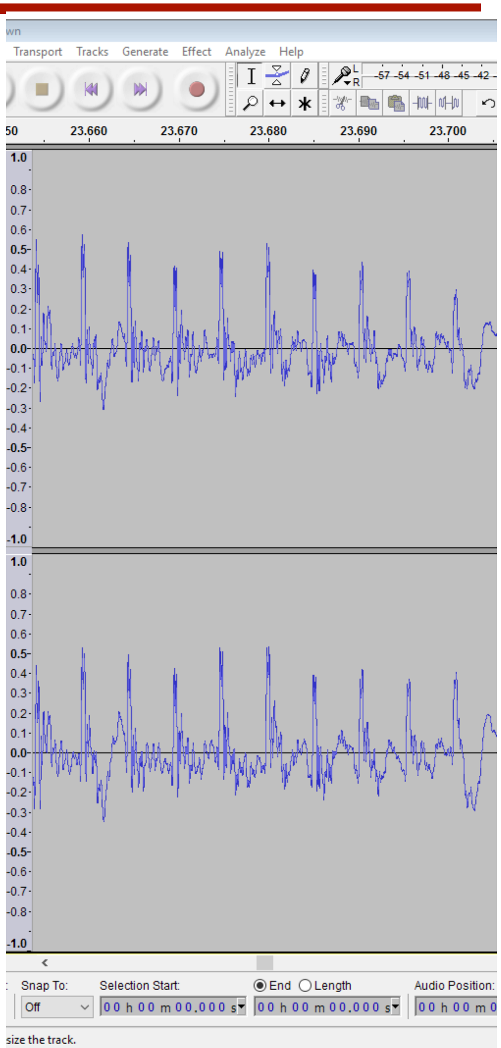


- Assume Vc=5V at t=0
- At t=0, switch flips down
- Plot Vc vs t

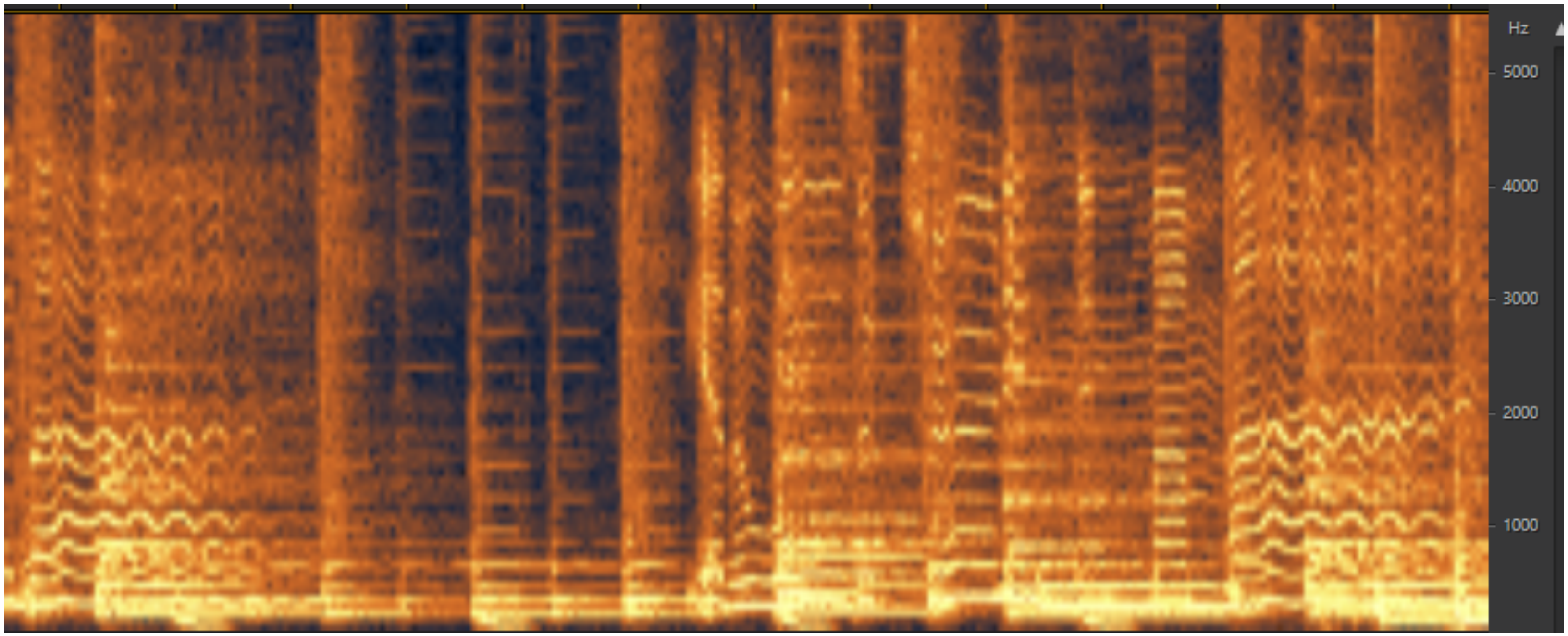
PREVIOUSLY IN E40M

Natural Way To Represent Sound

- We create pressure waves by moving a speaker
  - Larger voltage causes more deflection
- Sound both pushes and pulls the speaker cone
  - Voltages are both + and –
- Represent sound by voltage vs. time
- On computers
  - Sample voltage at 44K/sec
  - Digitize the voltage
    - Into 15 bit integer (signed)
  - Raw = .wav; compressed .mp3



Viewed In Terms of Sine-waves



- So you can take the music and look sinewaves in a block of time
  - Then repeat that for another block of time
  - This analysis plots frequency vs. time – a spectrogram

Formal Definition

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- Assuming a signal repeats every T seconds
  - Or we just have T seconds of data to look at …

$$v(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right)$$

- The term with n=1 is called the fundamental term
  - It is the lowest frequency that exist in a period of T
  - The other terms are called harmonics
    - They are integer multiples of the fundamental frequency $2\pi/T$

IMPEDANCE

Impedance

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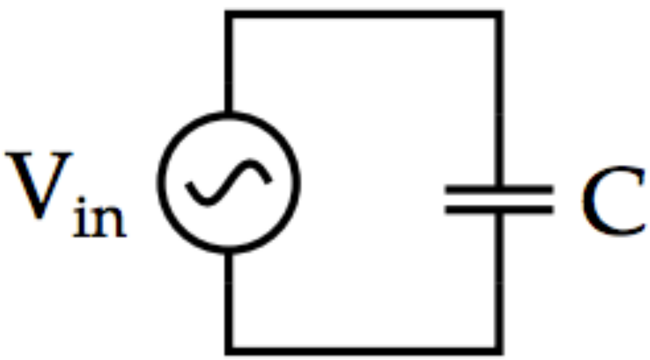
- Ratio of the sine wave voltage across the device
  - To the sine wave current through the device
- For resistors
  - This is just the resistance, and doesn't depend on freq.
- For capacitors
  - This is a well defined quantity

$$Z = \frac{1}{2\pi F\ C}$$

    - Which depends on the frequency of the sine wave

What happens when we put sine waves into a Capacitor?



- Vin = sin(2πf t)
- Ic = C dVin/dt
- Ic = C 2πf cos(2πf t)

- |Z| = Vin/Ic = 1/(2πf C)