Control of Manufacturing Processes

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MITMECHE Manufacturing

Laboratory for Manufacturing and Productivity

Topics for Today

- Physical Origins of Variation
  - Process Sensitivities
- Statistical Models and Interpretation
  - Process as a Random Variable(s)
  - Diagnosis of Problems
- Shewhart Charts
- Process Capability
- Next Steps: Optimization and Control

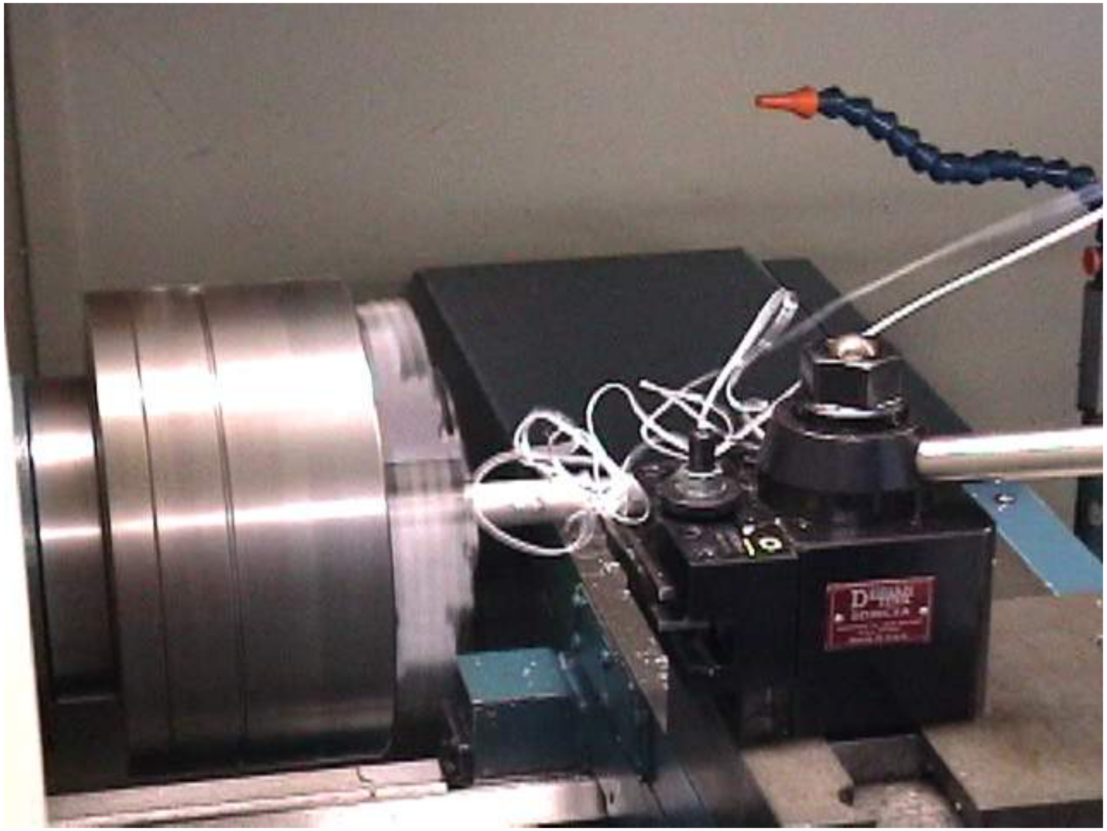
Process Objectives?

- Rate
- Quality
- Cost
- Flexibility

Process Control Objectives?

- Rate
- **Quality**
  - **Conformance to Specifications wrt**
    - **Geometry**
    - **Properties**
- Cost
- Flexibility

Lab Processes



CNC Turning

Critical Dimension:

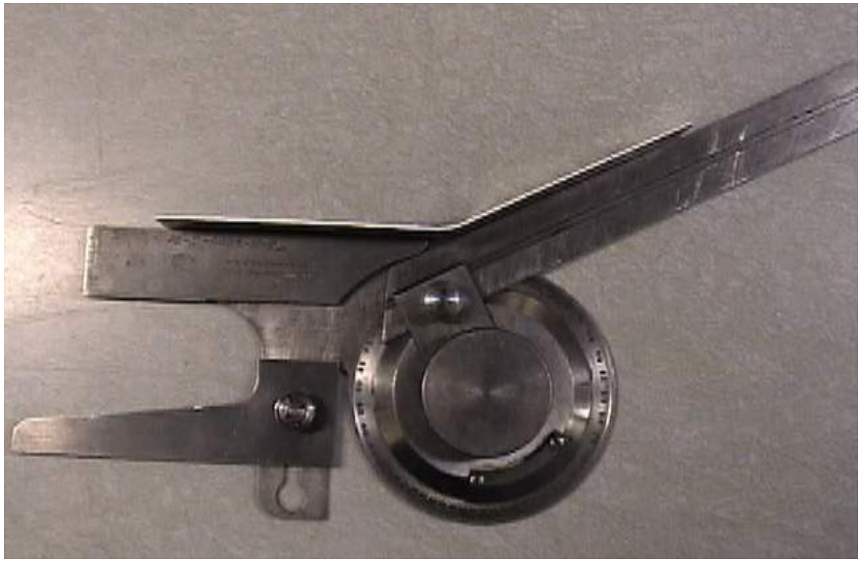
Shaft Diameter

Lab Processes

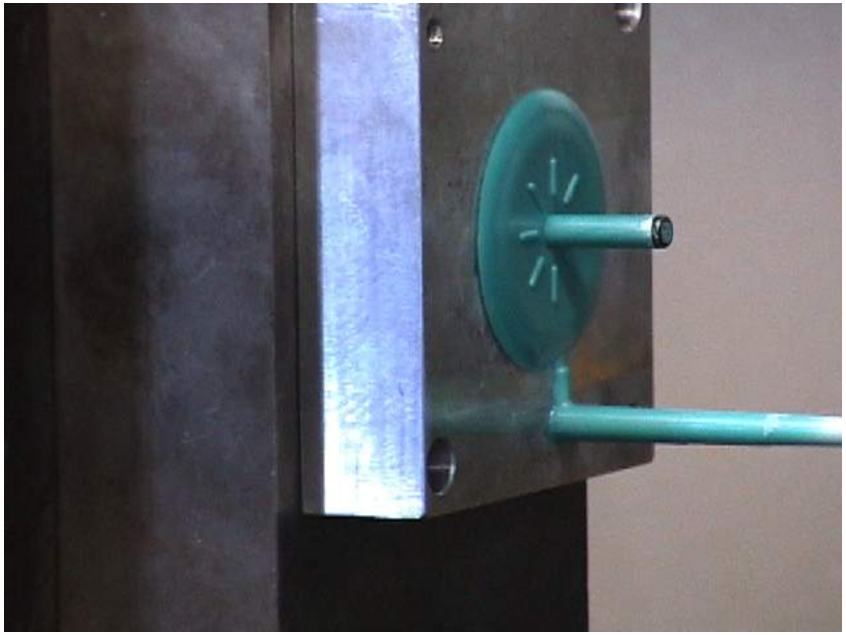
Brake Bending

Critical Dimension:

Part Angle

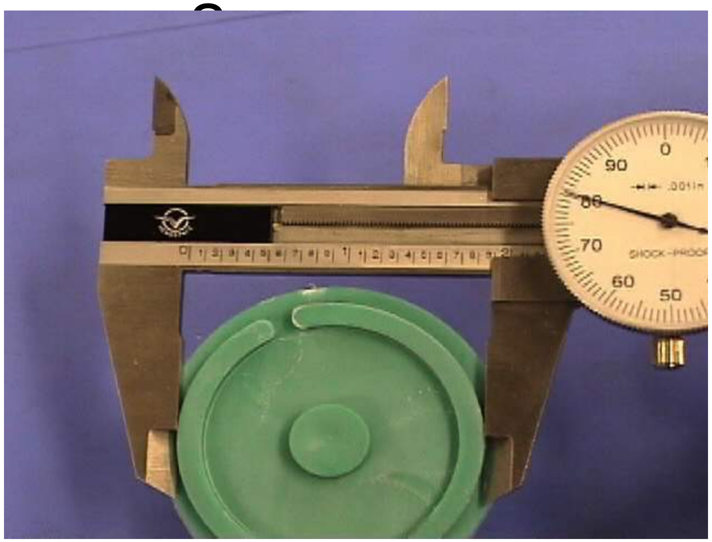


Lab Processes



Injection Molding

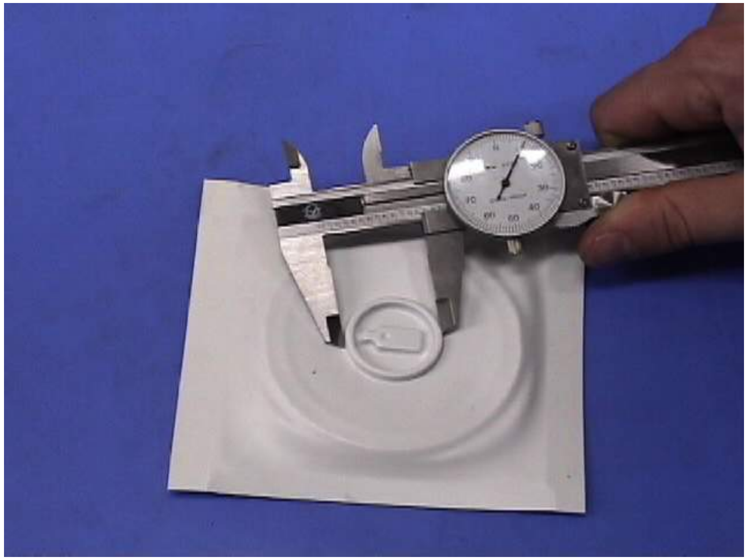
Critical Dimension:



Lab Processes

Thermoforming

Critical Dimension:



Other Related Problems: Cost, Rate and Flexibility:

- 100% inspection with high scrap rates
  - low throughput
  - high costs
- 100% Inspection with frequent rework
  - low throughput
  - high costs
- High Variability at changeover
  - Reluctance to changeover
  - low flexibility

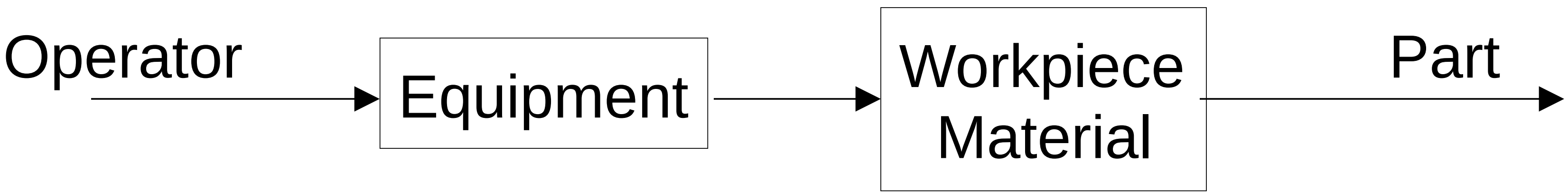
Manufacturing Processes

- How are they defined?
- How to they do their thing?
- How can they be categorized?

- **Why don’t they always get it right?**

Origins of Variation

The Process Components



- Etch bath
- Injection Molder
- Lathe
- Draw Press
- ...

- Coated Silicon
- Plastic Pellets
- Bar stock
- Sheet Metal
- ...

- Semiconductor
- Connector Body
- Shaft
- Hood
- ...

What Causes *Variation* in the Process Output?

- Material Variations
  - Intrinsic Properties, Initial Geometry
- Equipment Variations
  - Non-repeatable, long term wear, deflections
- Operator Variations
  - Inconsistent control, excessive “tweaking”
- “Environment” Variations
  - Temperature and Handling inconsistencies

Can We Rank These?

- Likelihood of Variation?
- Frequency of Variation?
- Magnitude of Variation?
- Sensitivity to Variation?

Can We Rank These?

- Equipment
  - Fixed "Iron"
  - Can be Automated (Controlled) to Keep Energy States as Desired
- Material
  - "Flows" Through the Process
    - Constantly Changing
  - Energy Transfer from Equipment Variable

Process Control Hierarchy

- **Identify and Reduce Disturbances**
  - Good Housekeeping (Ops Management)
  - Standard Operations (SOP’s)
  - **Statistical Analysis and Identification of Sources**
  - **Feedback Control of Machines**
  - Reduce Sensitivity (Process Optimization or Robustness)
  - **Measure Sensitivities via Designed Experiments**
  - Adjust “free” parameters to minimize
- Measure output and manipulate inputs
  - **Feedback control of Output(s)**

Why not Always "Process Output Control"?

- Lack of Measurements
  - Shape not accessible
- Lack of Spatial Resolution
  - Complex shape, simple control u
- Cost/Benefit vs. Other Methods
- Sufficiency of Equipment Control
  - e.g. numerical control

Modeling Variation

Applying Statistics to Manufacturing: The Shewhart Approach (circa 1925)*

- All Physical Processes Have a Degree of Natural Randomness
- A Manufacturing Process is a Random Process if all "Assignable Causes" (identifiable disturbances) are eliminated
- A Process is "In Statistical Control" if only "Common Causes" (Purely Random Effects) are present.

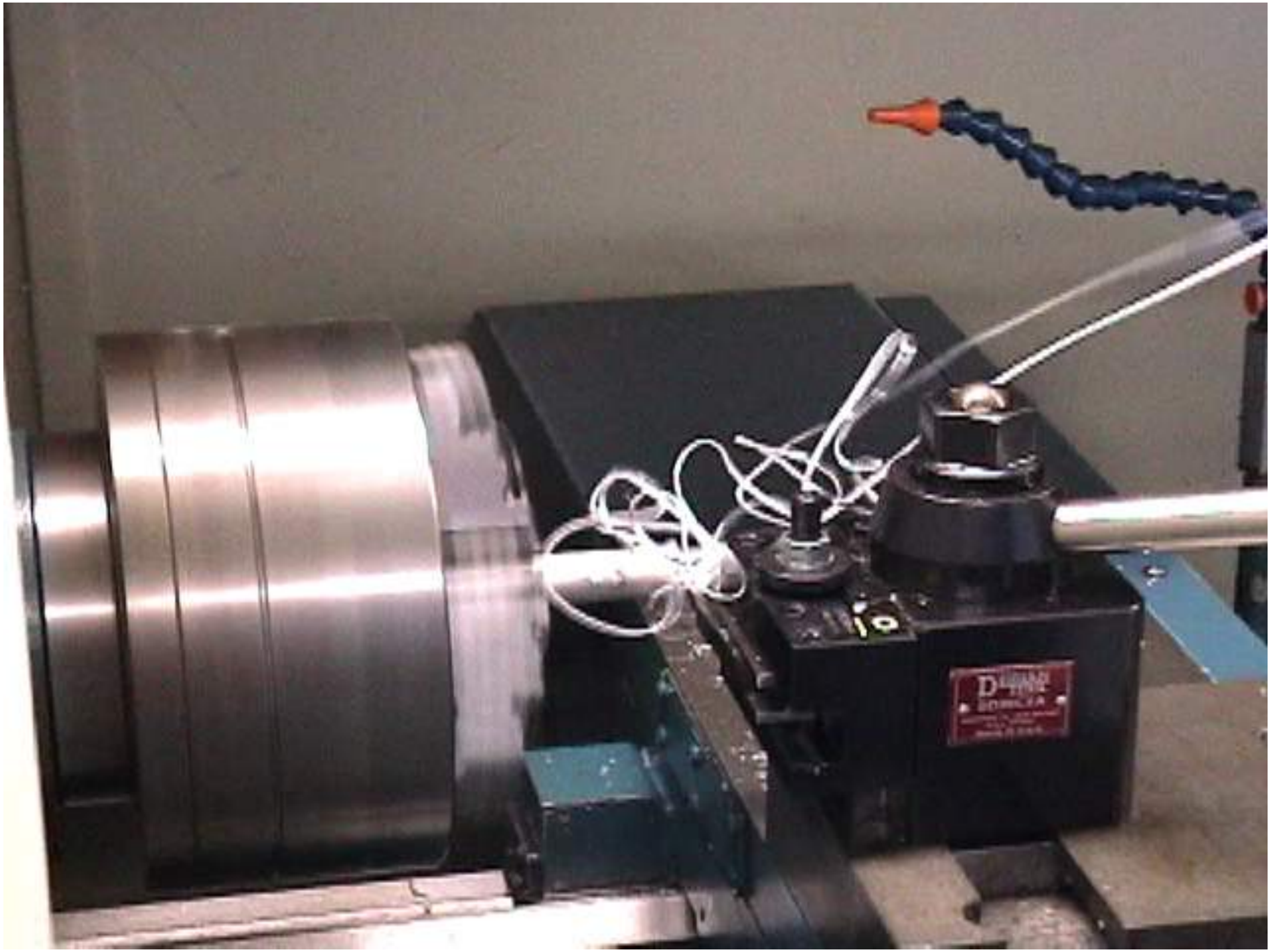
W.A. Shewhart, "The Applications of Statistics as an Aid in Maintaining Quality of a Manufactured Product", Journal of the American Statistical Association, 20, No.. 152, Dec. 1925.

Shewhart Applied to Manufacturing

- Measure and Plot the Process Output
- Look for Any Sign of Non-Random (Deterministic) Behavior
  - No in Statistical Control
- Identify the Cause of that Behavior and Reduce or Eliminate it
- Verify That the Process is Now Purely Random
  - In Statistical Control

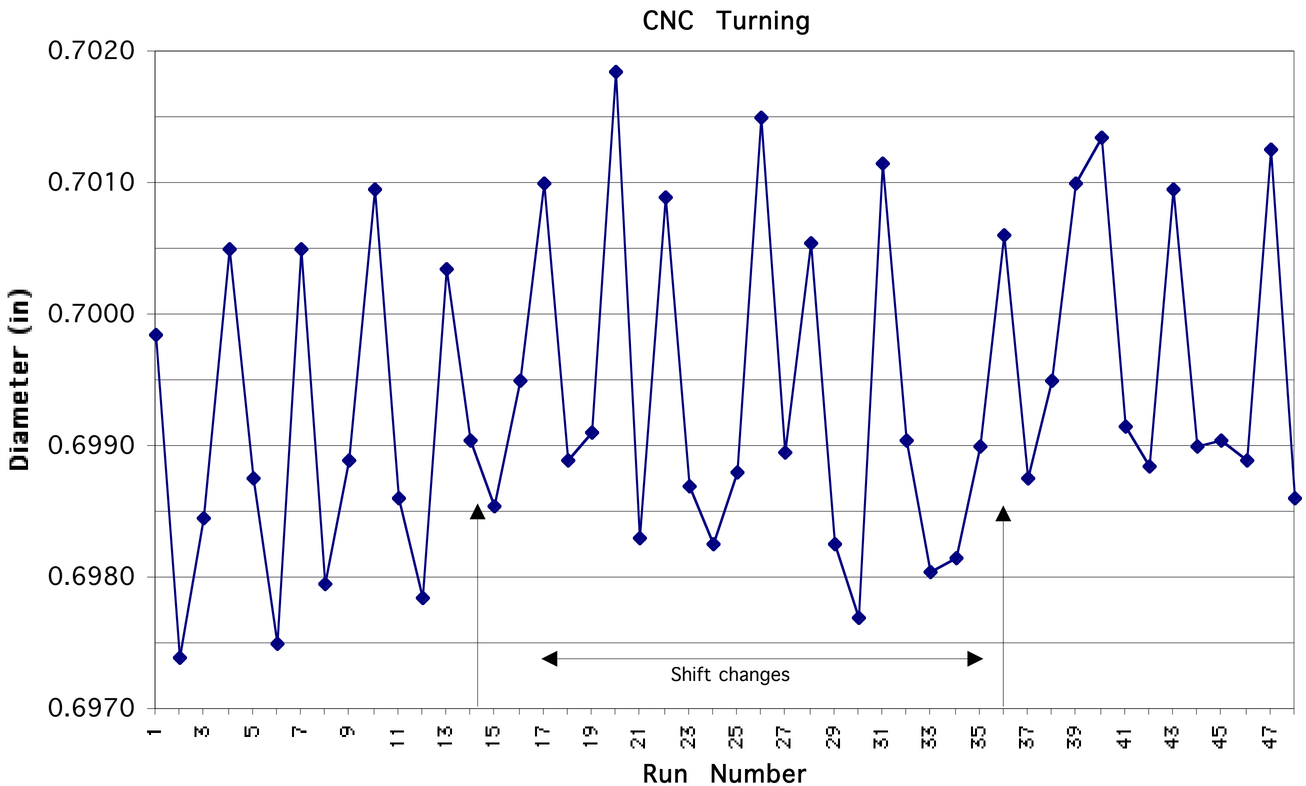
Statistical Models for Manufacturing

Consider: Turning Process



Observations from Experiments

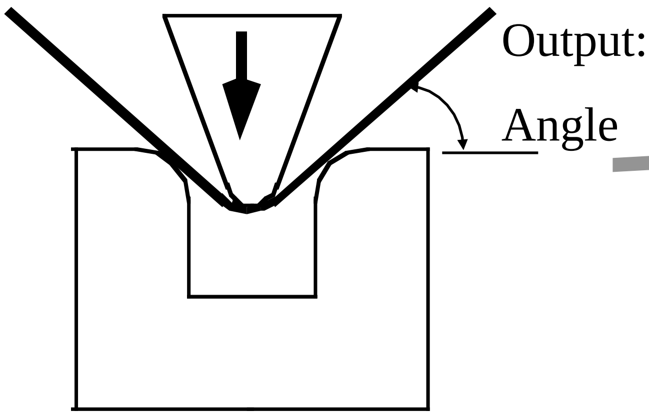
- Randomness + Deterministic Changes

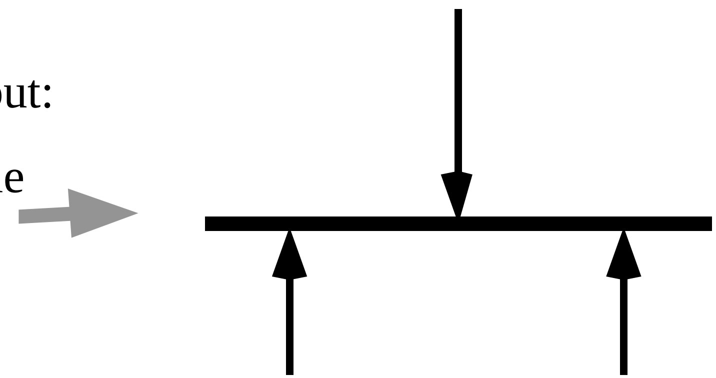


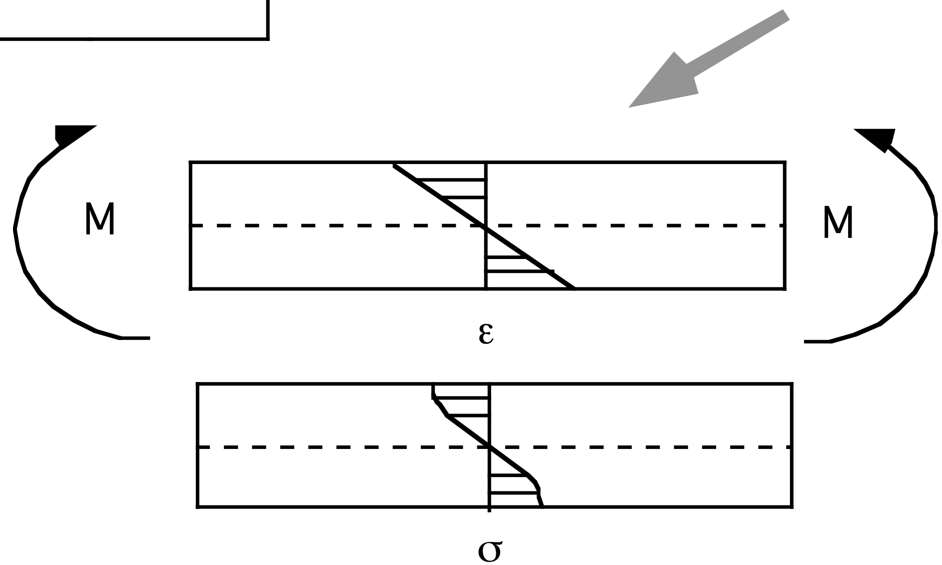
random or unknown

$\Delta\alpha$

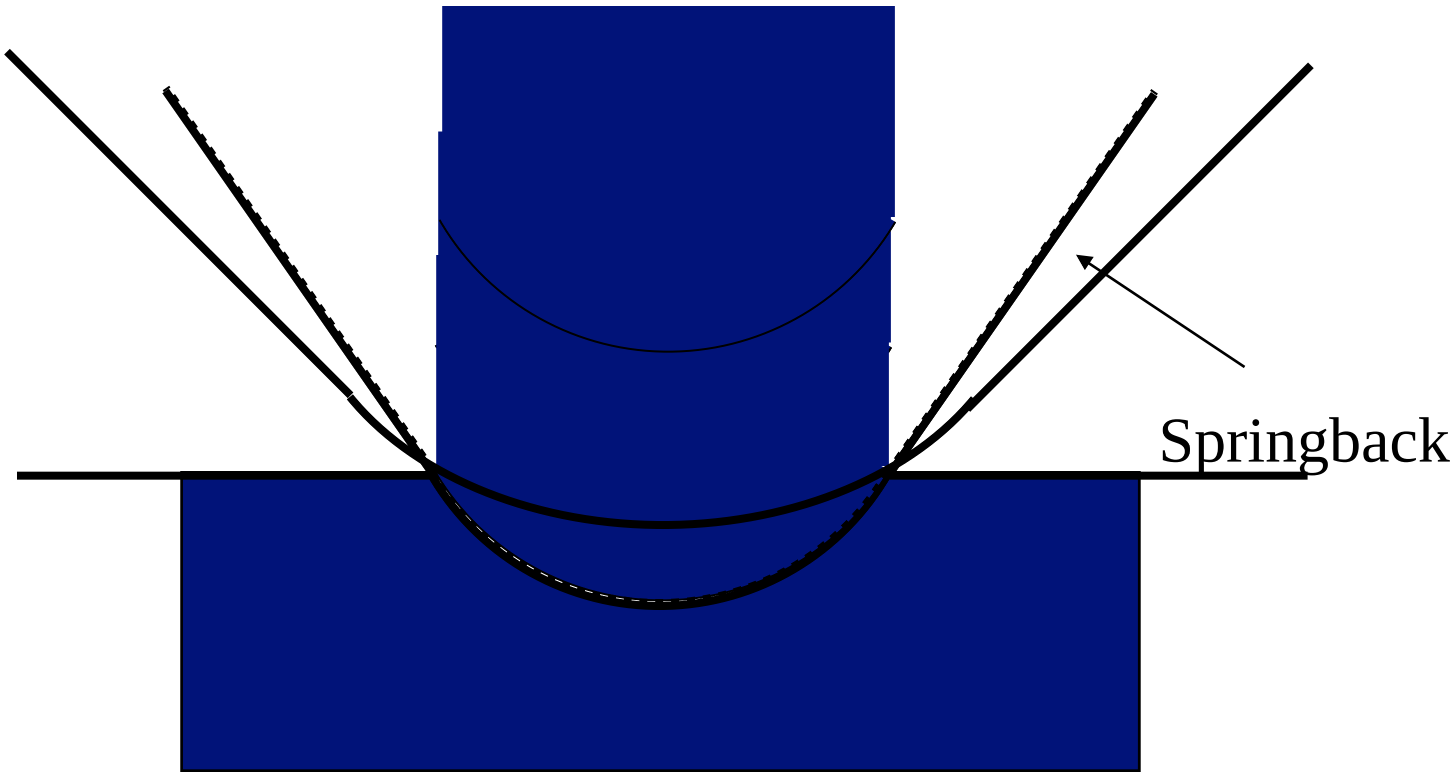
Brake Bending of Sheet





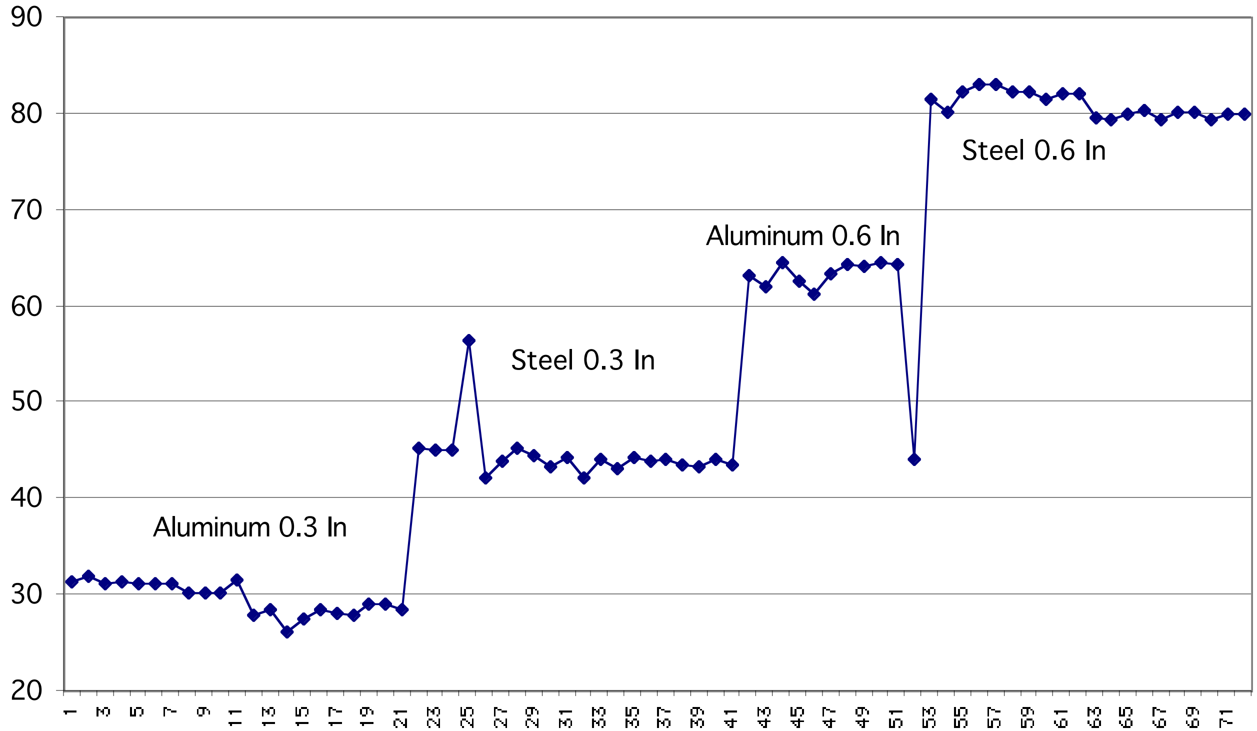


Bending Process



Observations from Bending Process

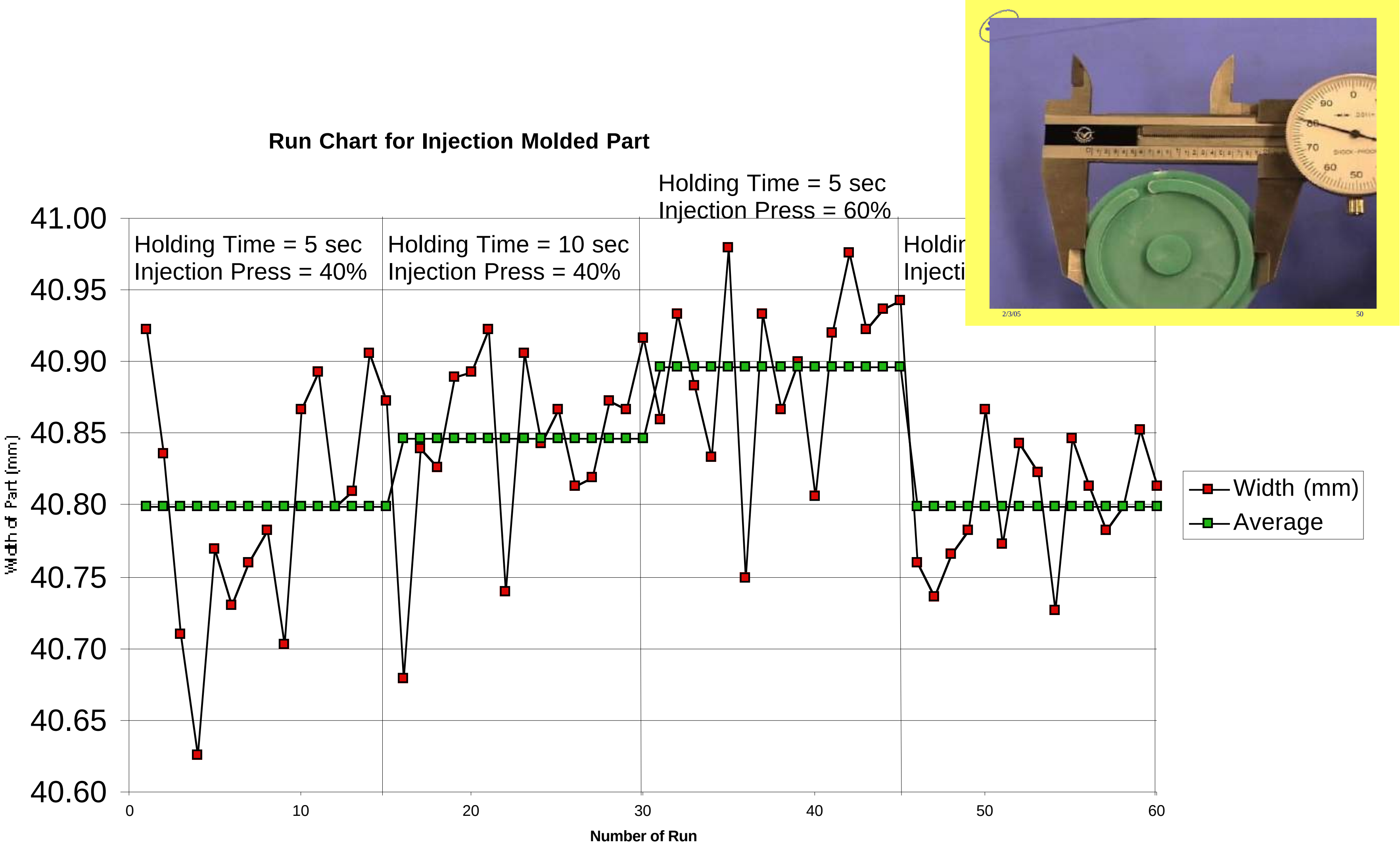
- Clear Input-Output Effects (Deterministic)
- Also Randomness as well



Angle changes with depth

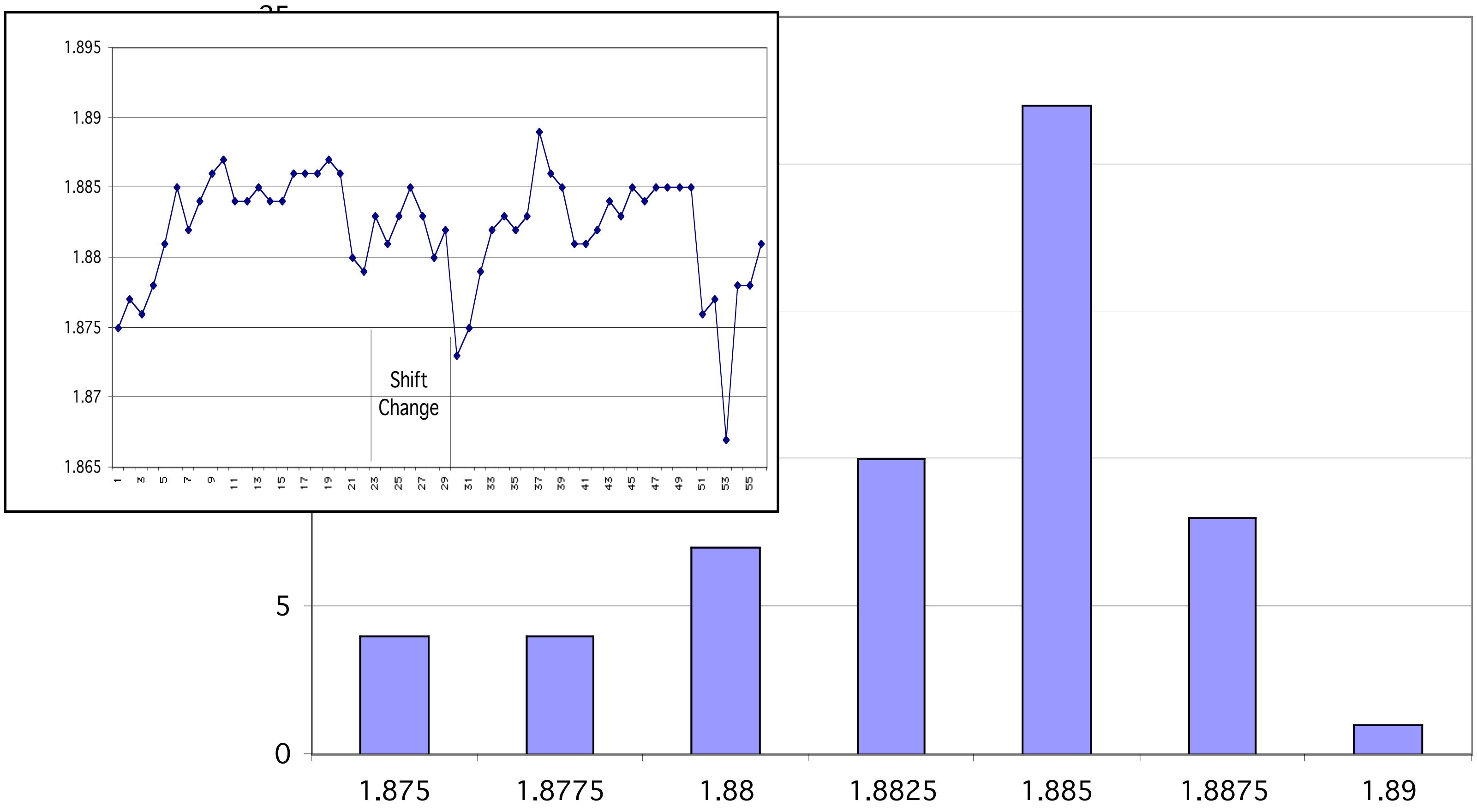
$\Delta Y \rightarrow \Delta u$

Observations from Injection Molding

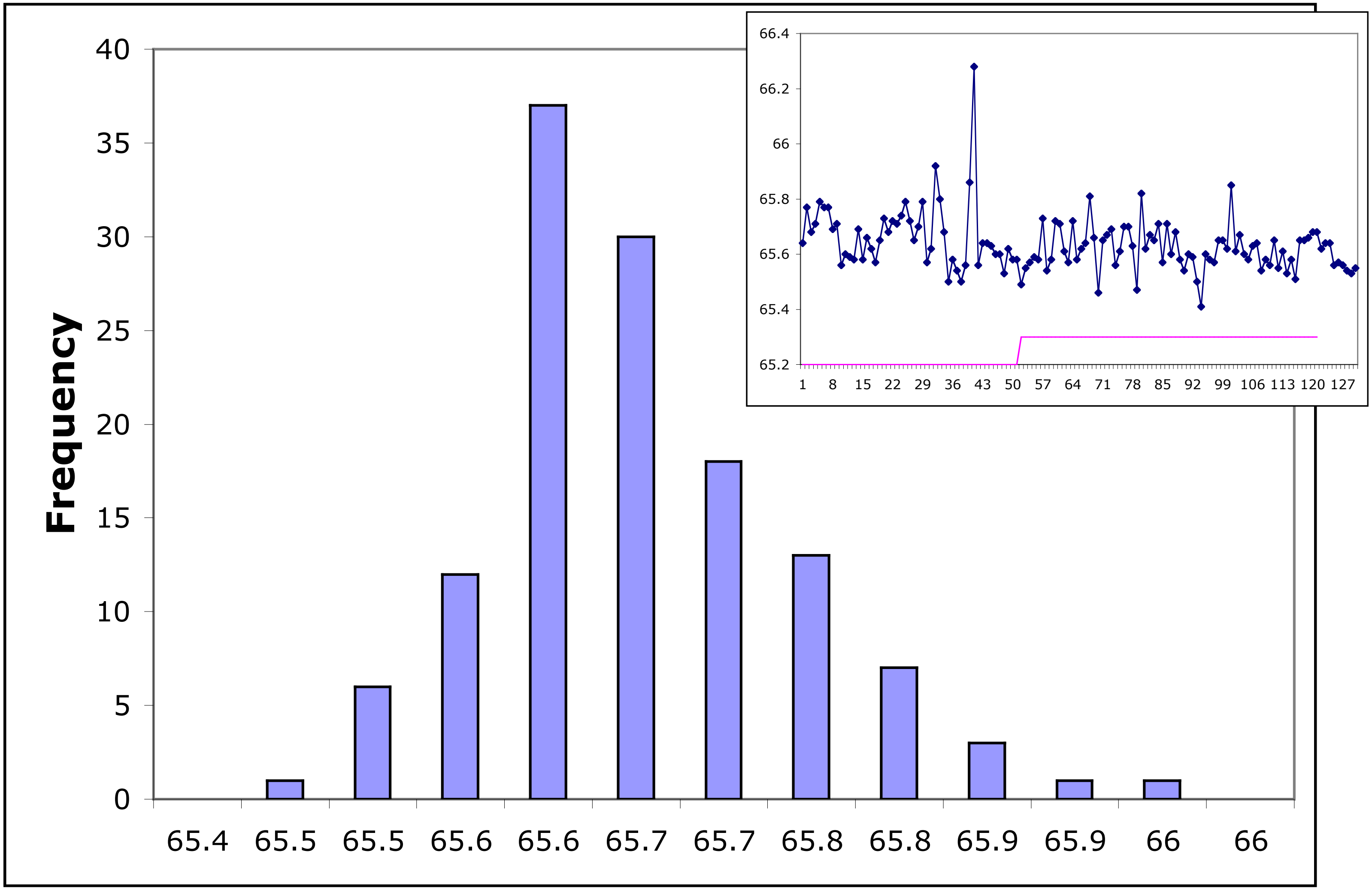


Consider:
No Effective Changes ($\partial Y/\partial u = 0$)

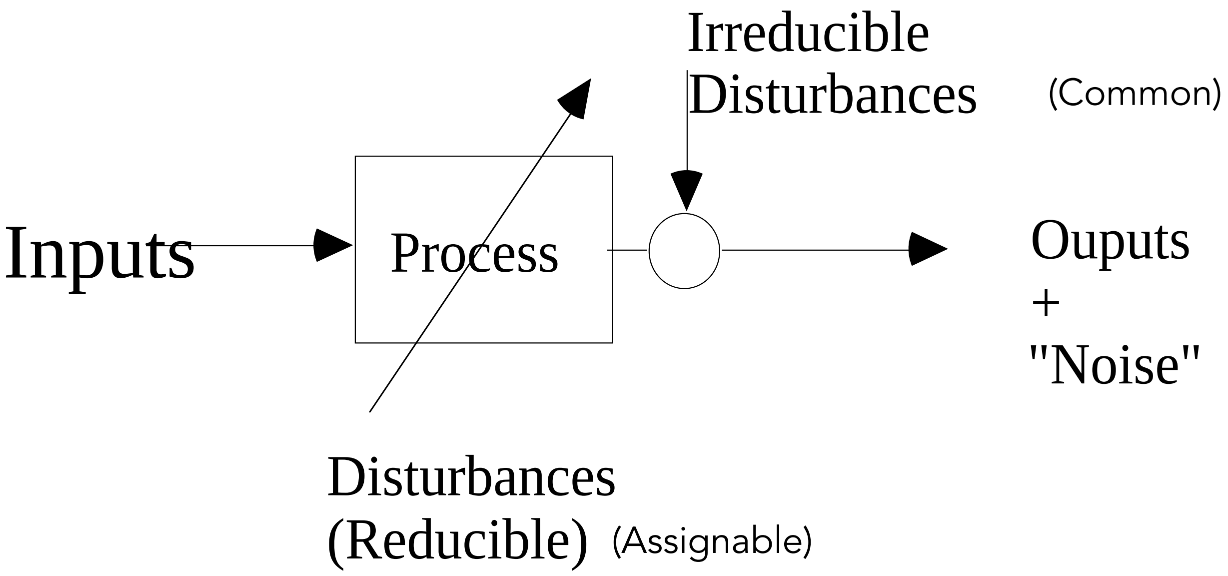
- Injection Molding Entire Run



Injection Molding (S' 2003)



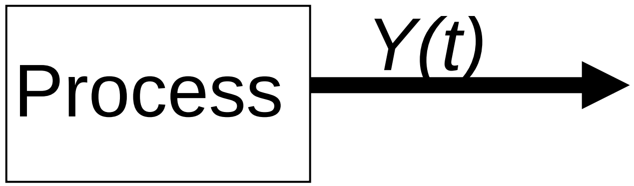
How To Model
to Distinguish these Effects?



A Random Process + A Deterministic Process

Random Processes

- Consider the Output-only, "Black Box" view of the Run Chart

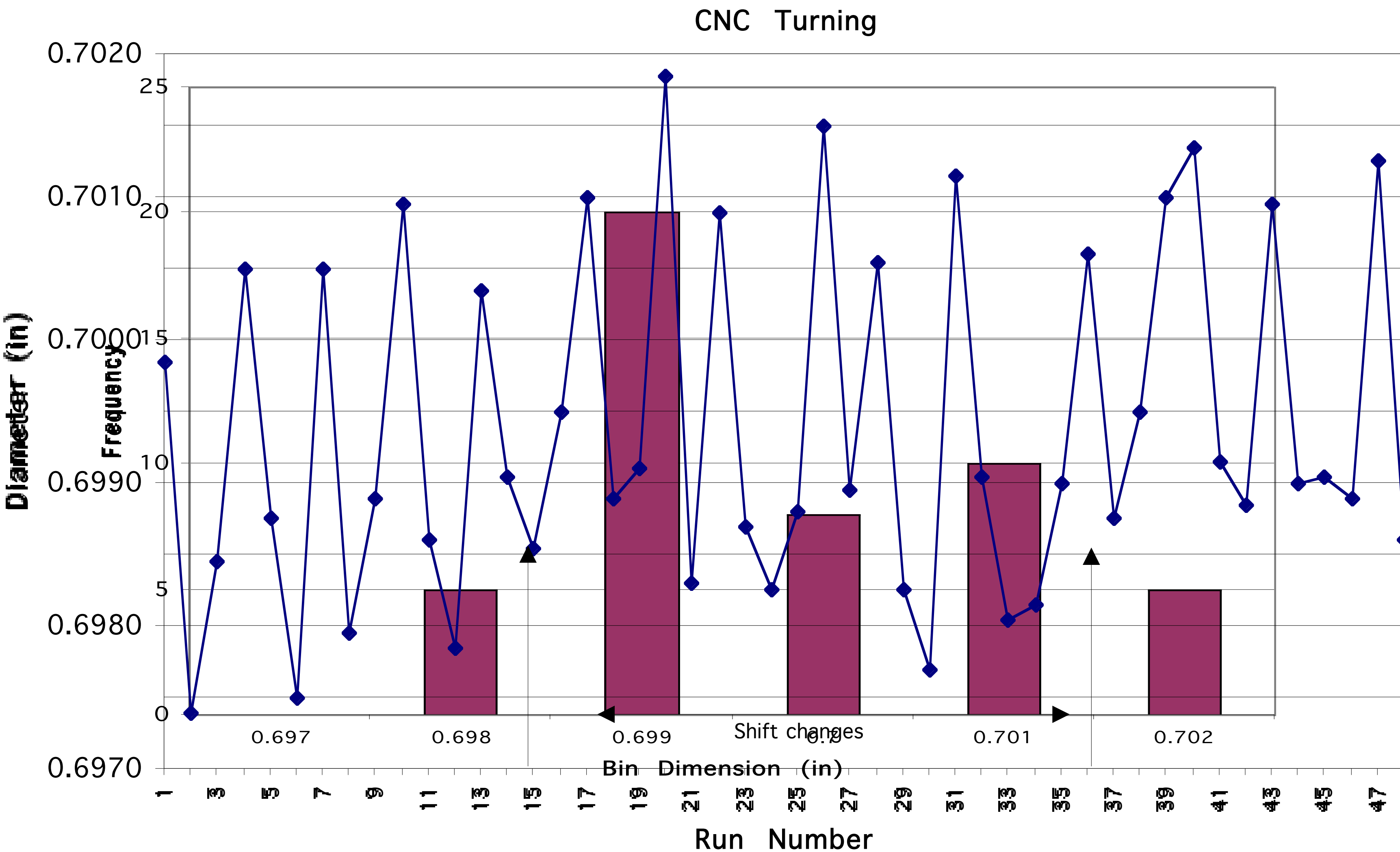


- How do We Characterize The Process?
  - Using $Y(t)$ only
- WHY do we Characterize the Process
  - Using $Y(t)$ only?

How to Describe Randomness?

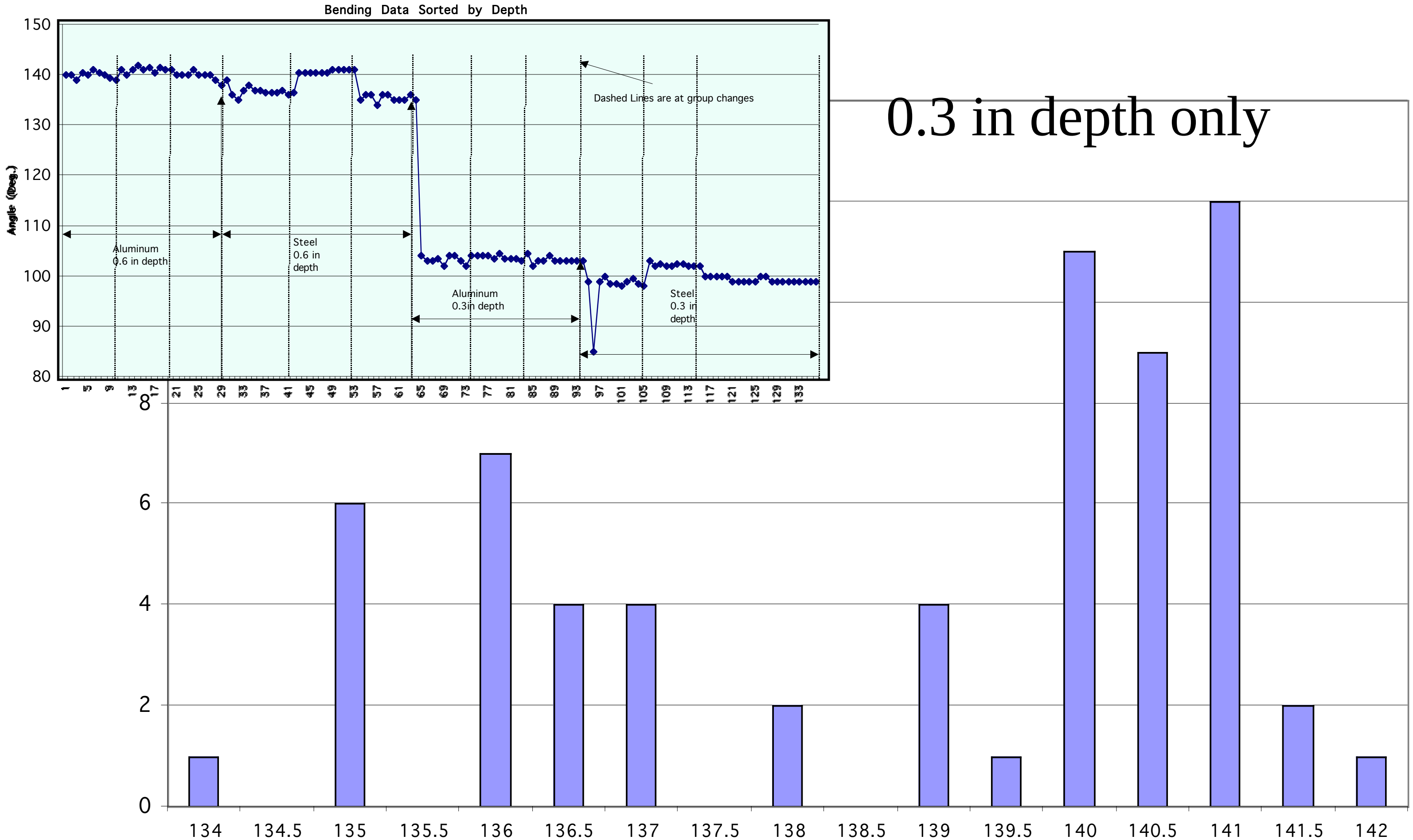
- Look at a **Frequency Histogram** of the Data
- Estimates likelihood of certain ranges occurring:
  - $Pr(y_1 < Y < y_2)$
  - "Probability that a random variable Y falls between the limits y_1 and y_2"

Histogram for CNC Turning



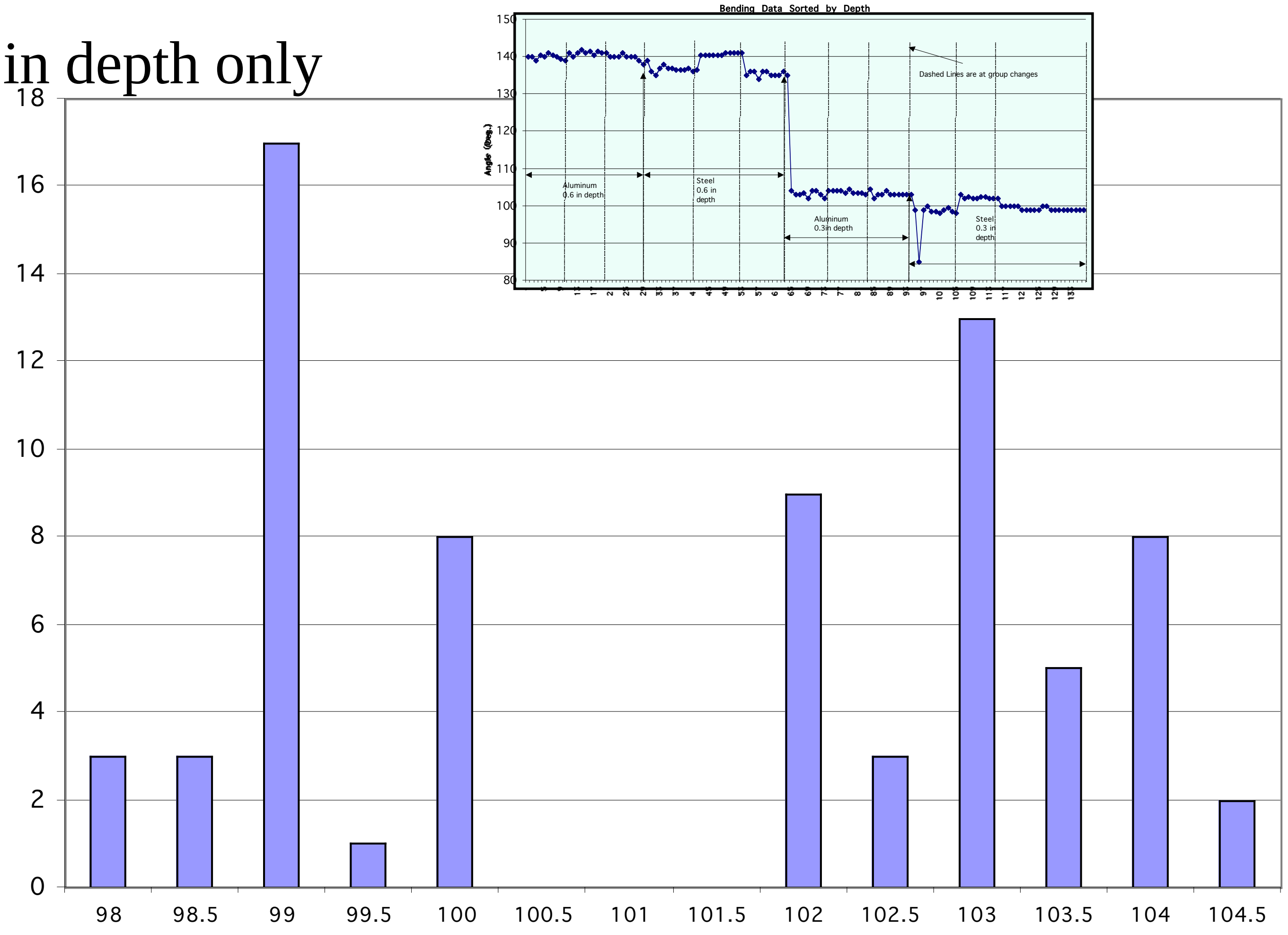
Histogram for Bending

(MIT 2002 data)



Histogram for Bending
(MIT 2002 data)

0.6 in depth only



Conclusion?

- When there are no input change (e.g. using SOP's) a consistent histogram pattern *can* emerge
- How do we use knowledge of this pattern?
  - – Predict behavior
  - – Set limits on "normal" behavior
- *Define analytical probability density functions*

Analysis of Histograms

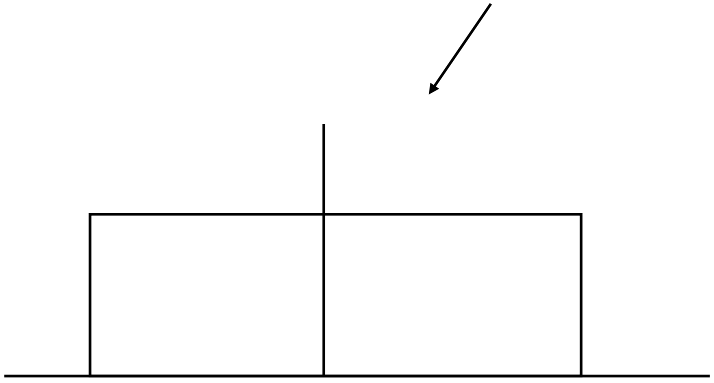
- Is there a consistent pattern?
- Is an underlying “parent” distribution suggested?

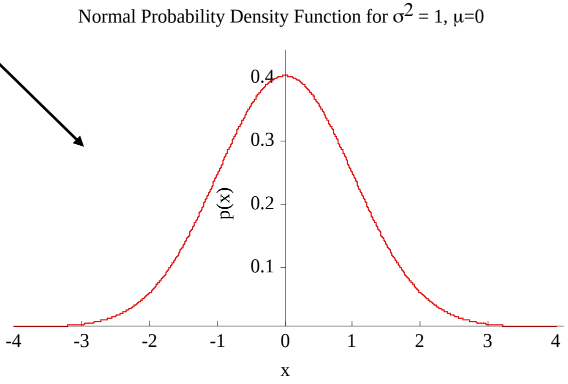
Underlying or "Parent" Probability

- A model of the "true", continuous behavior of the random process
- Can be thought of as a continuous random variable obeying a set of rules (the *probability function*)
- We can only glimpse into these rules by sampling the random variable (i.e. the process output)
- Underlying process can have
  - Continuous Values (e.g. geometry)
  - Discrete Values (e.g. defect occurrence)

Process Outputs as a Random Variable

- The Histogram suggests a *pdf*
  - Parent or underlying behavior "sampled" by the process
- Standard Forms (There are many)
  - e.g. The Uniform and Normal pdf's

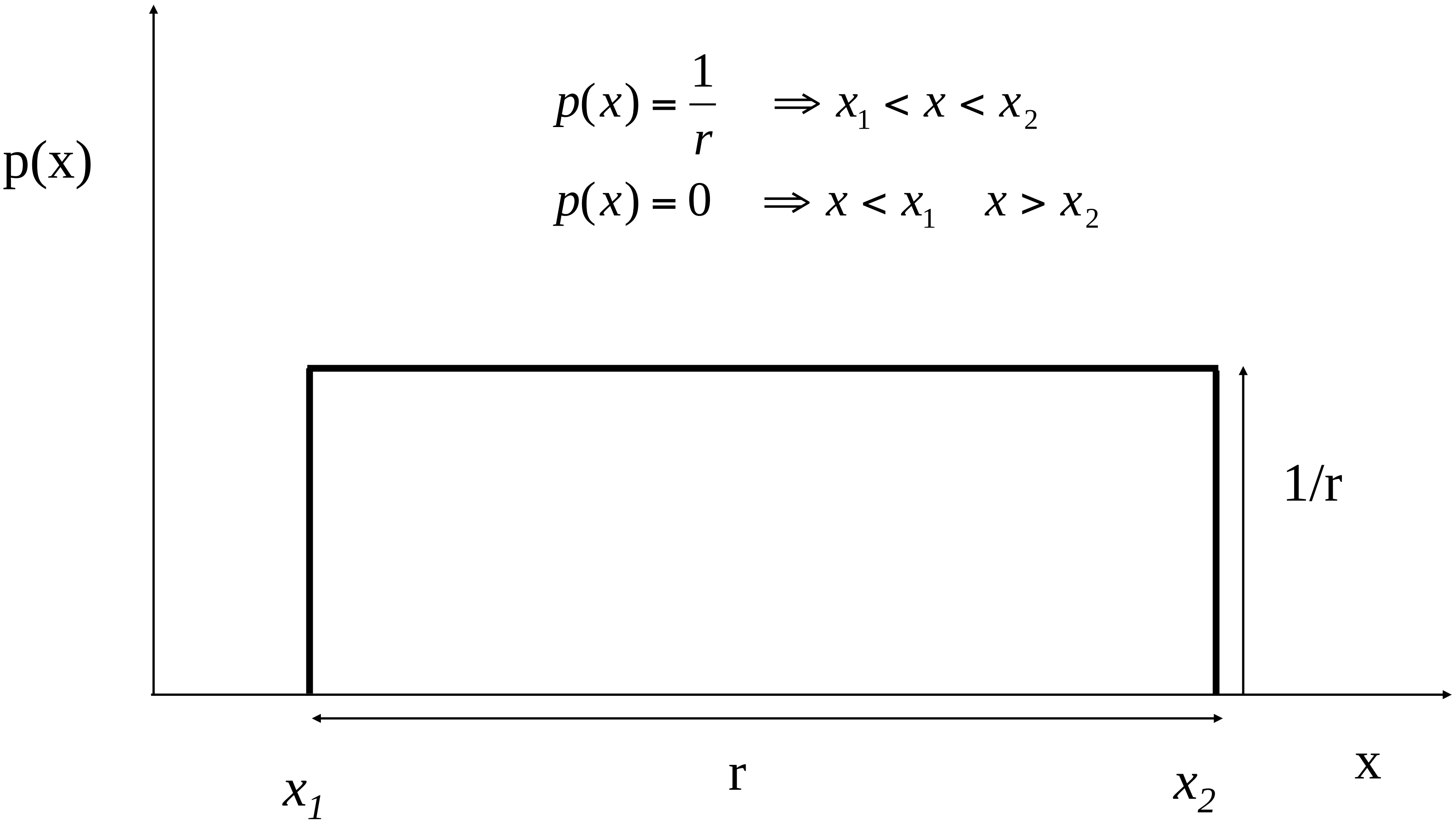




The Uniform Distribution

$$p(x) = \frac{1}{r} \quad \Rightarrow x_1 < x < x_2$$

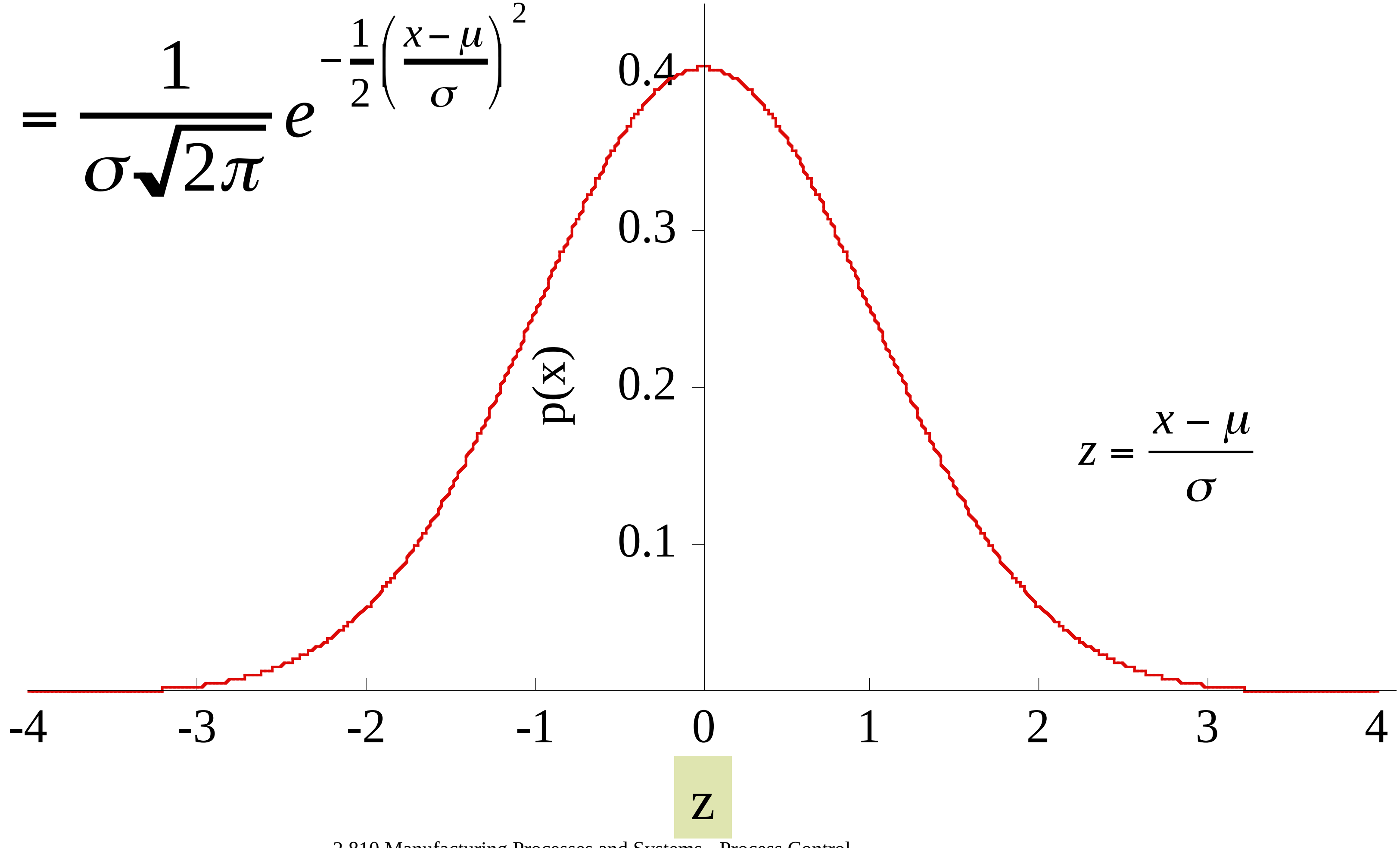
$$p(x) = 0 \quad \Rightarrow x < x_1 \quad x > x_2$$



Standard Normal Distribution

Normal Probability Density Function for $\sigma^2 = 1$, $\mu=0$

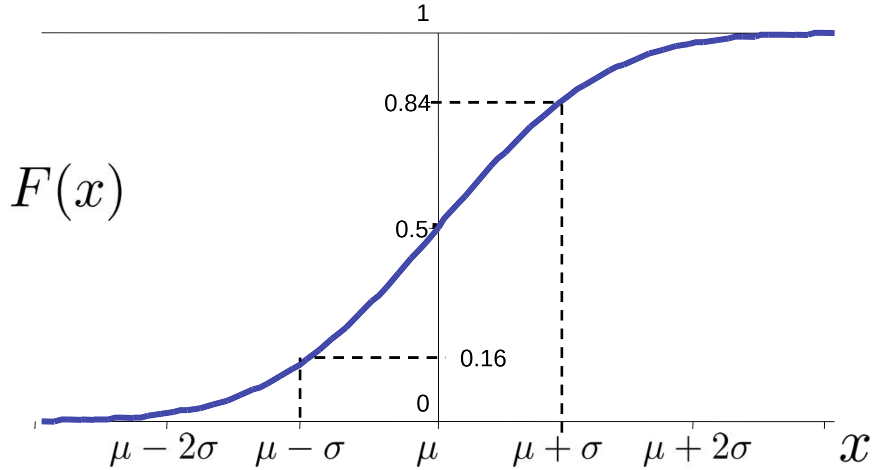
$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



$$z = \frac{x-\mu}{\sigma}$$

Continuous Distribution: Normal or Gaussian

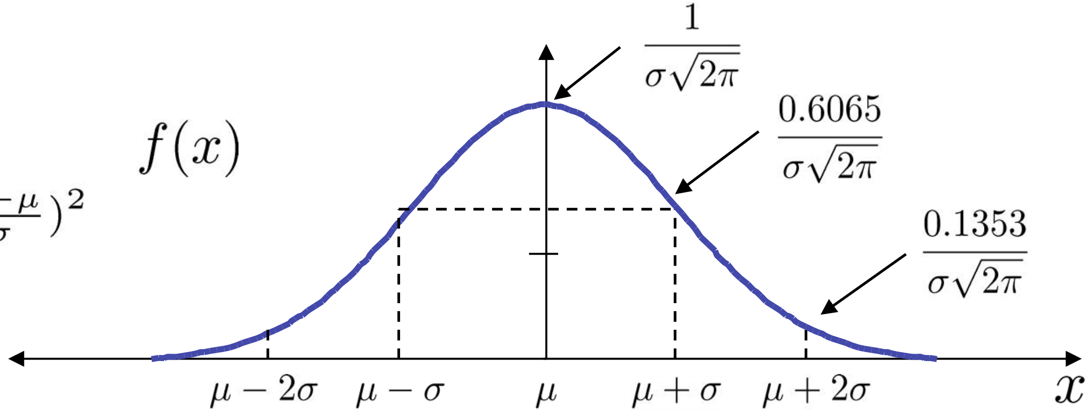
cdf



pdf

$$x \sim \mathrm{N}(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



Properties of the Normal pdf

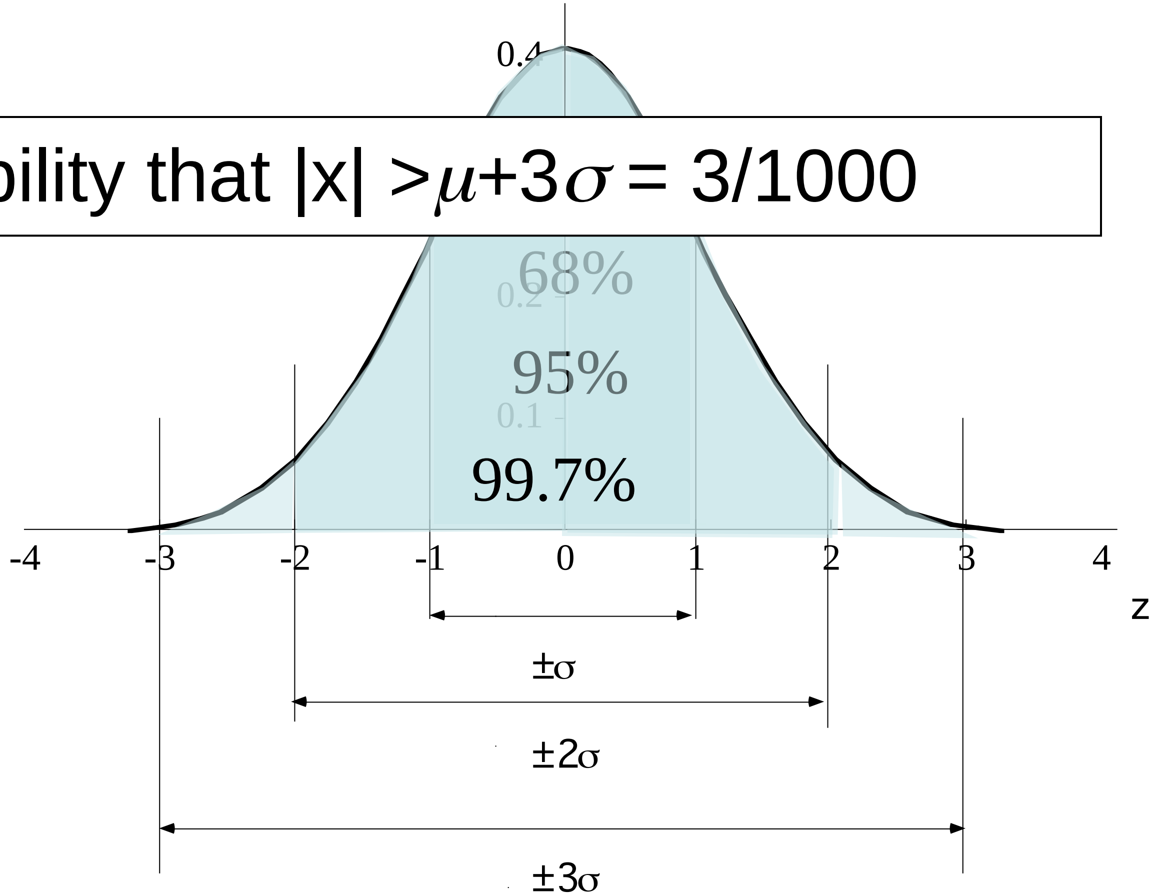
- Symmetric about mean
- Only two parameters:

  μ and σ^2

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

- Superposition Applies:
  - sum of normal random variables has a normal distribution

Interpretation of the PDF: Confidence Intervals



- Probability that |x| >μ+3σ = 3/1000

Model Calibration

- For the Normal PDF, we need two parameters: μ ***and*** σ
- We have to **estimate** μ *and* σ using sample statistic based on samples of the output (i.e. measurements)

Sample Statistics

$$x(j) = \text{samples of } x(t) \text{ taken } n \text{ times}$$

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x(j): \quad \text{Average or Sample Mean}$$

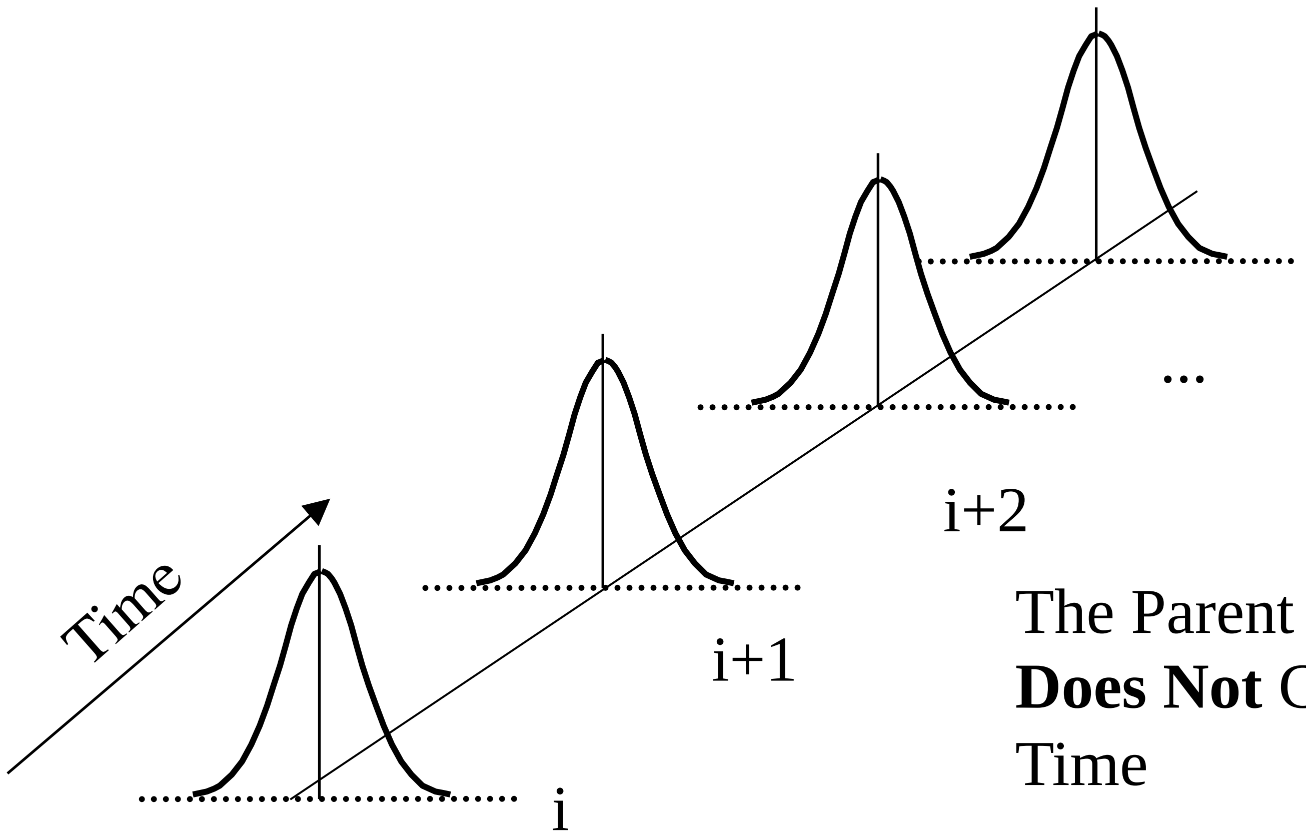
$$S^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x(j) - \bar{x})^2 : \quad \text{Sample Variance}$$

$$S = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x(j) - \bar{x})^2} : \quad \text{Sample Std.Dev.}$$

Conclusions

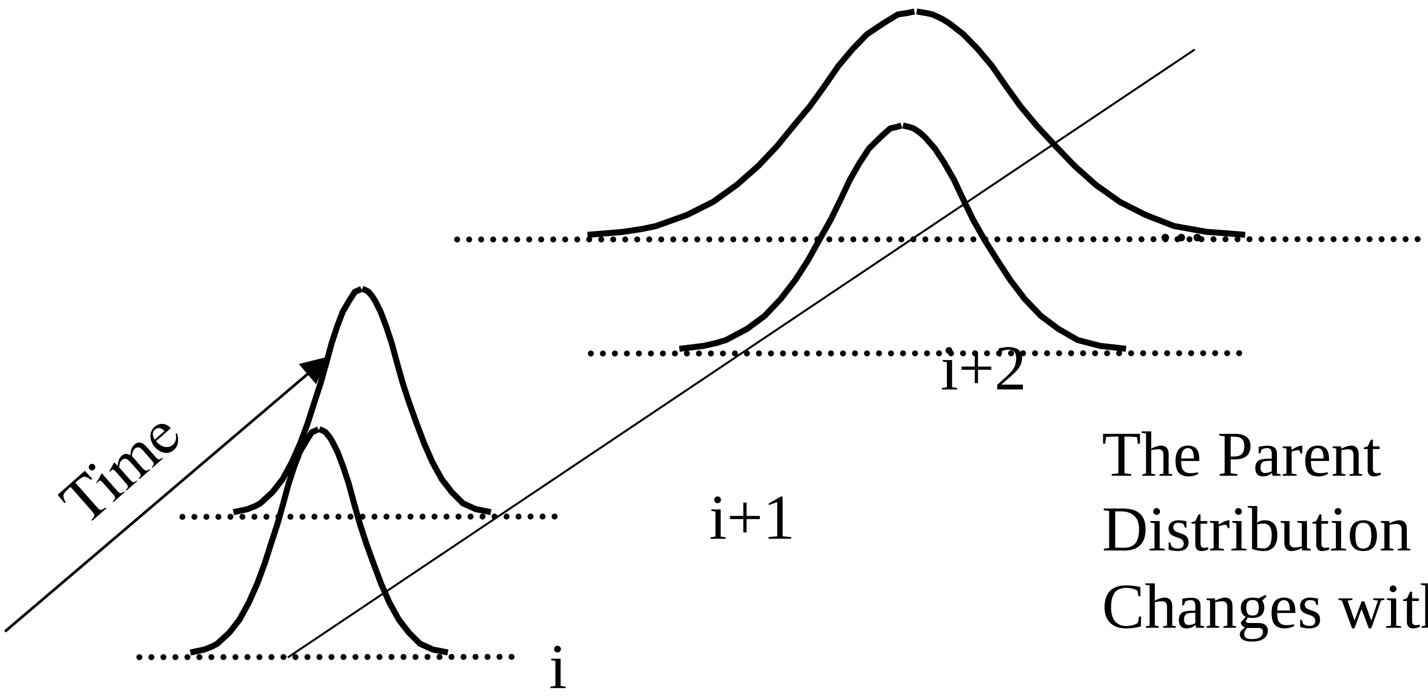
- All Physical Processes Have a Degree of **Natural Randomness**
- We can Model this Behavior using **Probability Distribution Functions**
- We can **Calibrate** and Evaluate the Quality of this Model from Measurement Data using appropriate **Sample Statistics**

"In-Control"



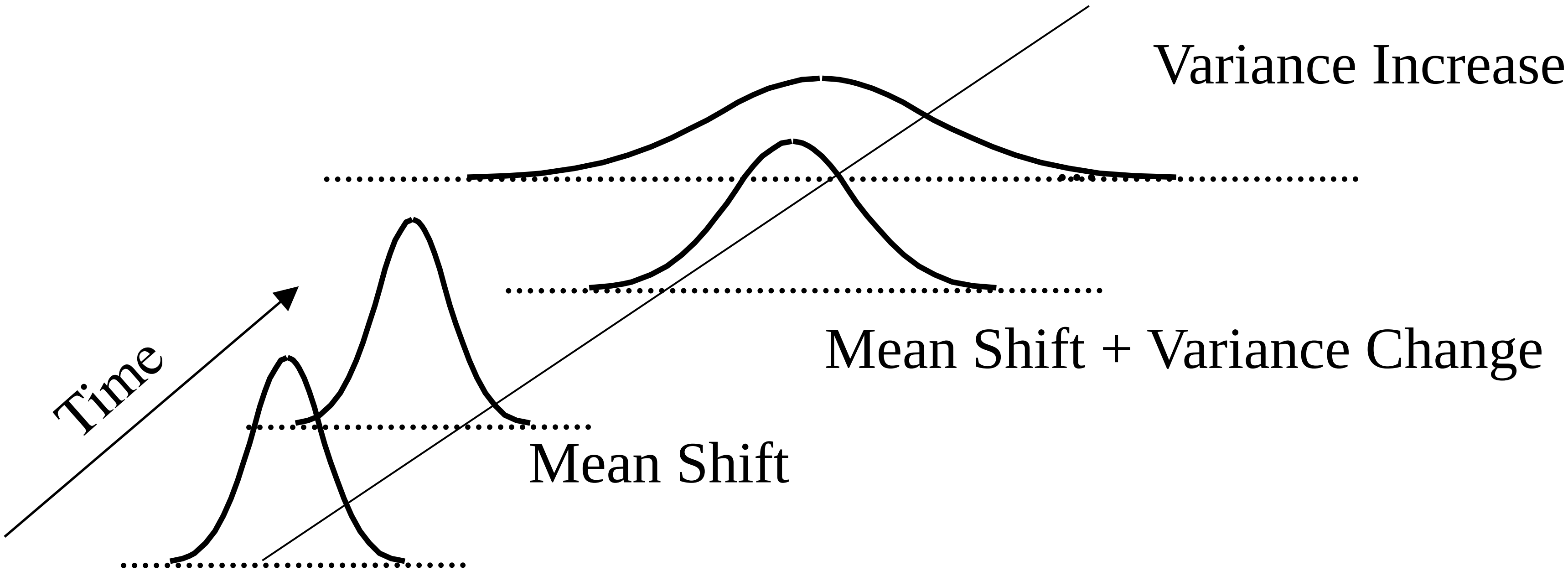
The Parent Distribution **Does Not** Change with Time

"Not In-Control"

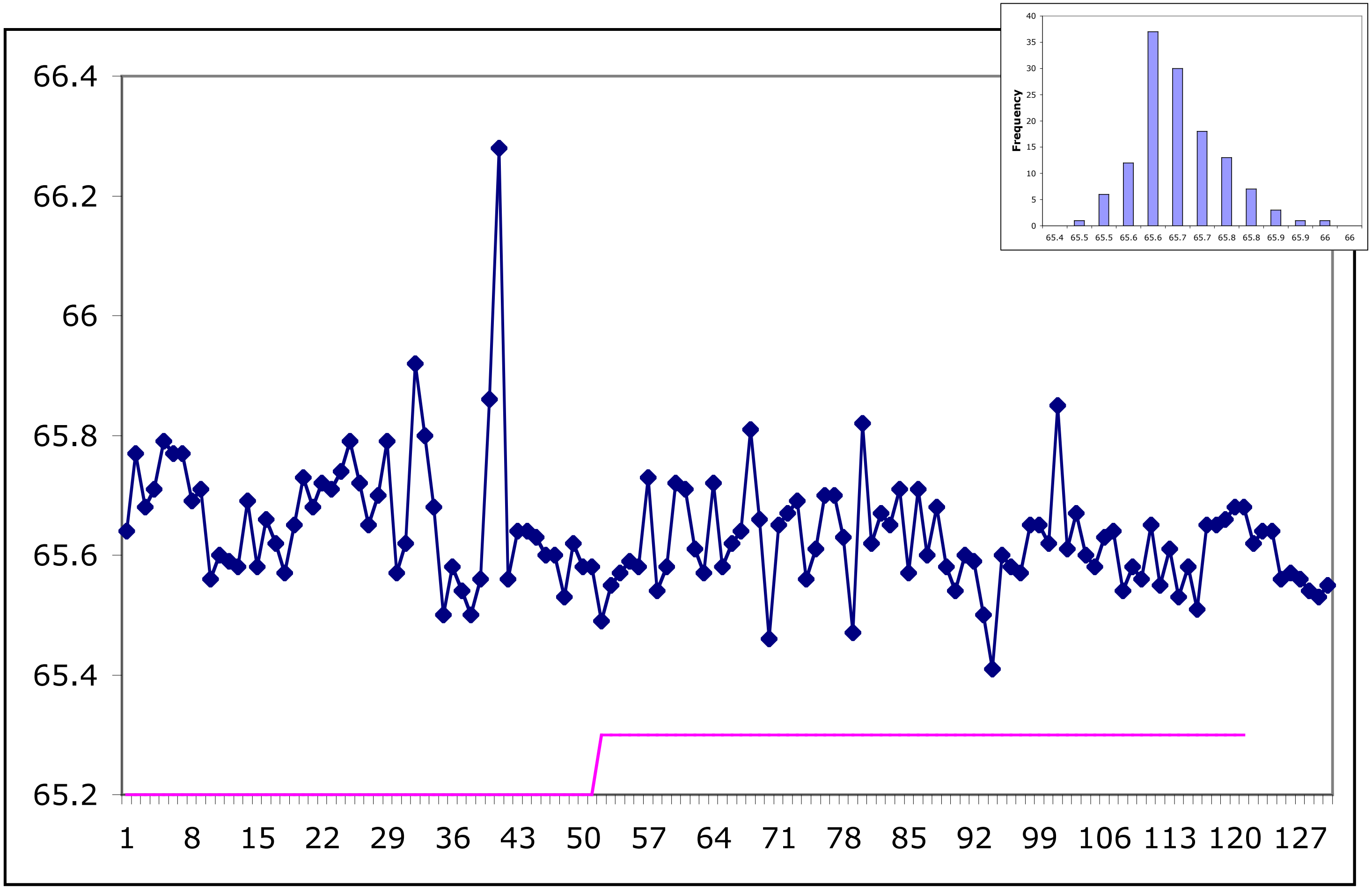


The Parent Distribution Changes with Time

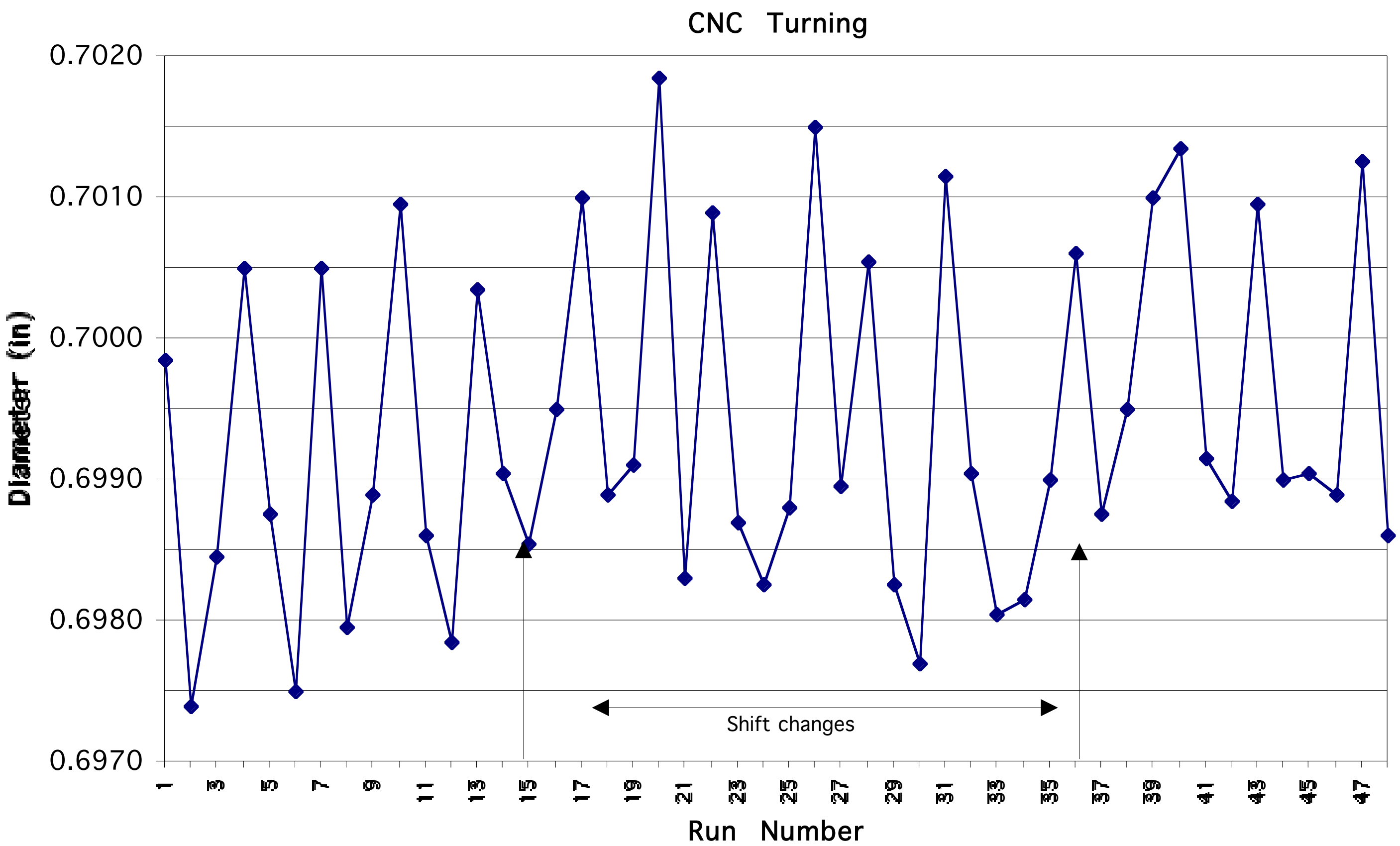
"Not In-Control"



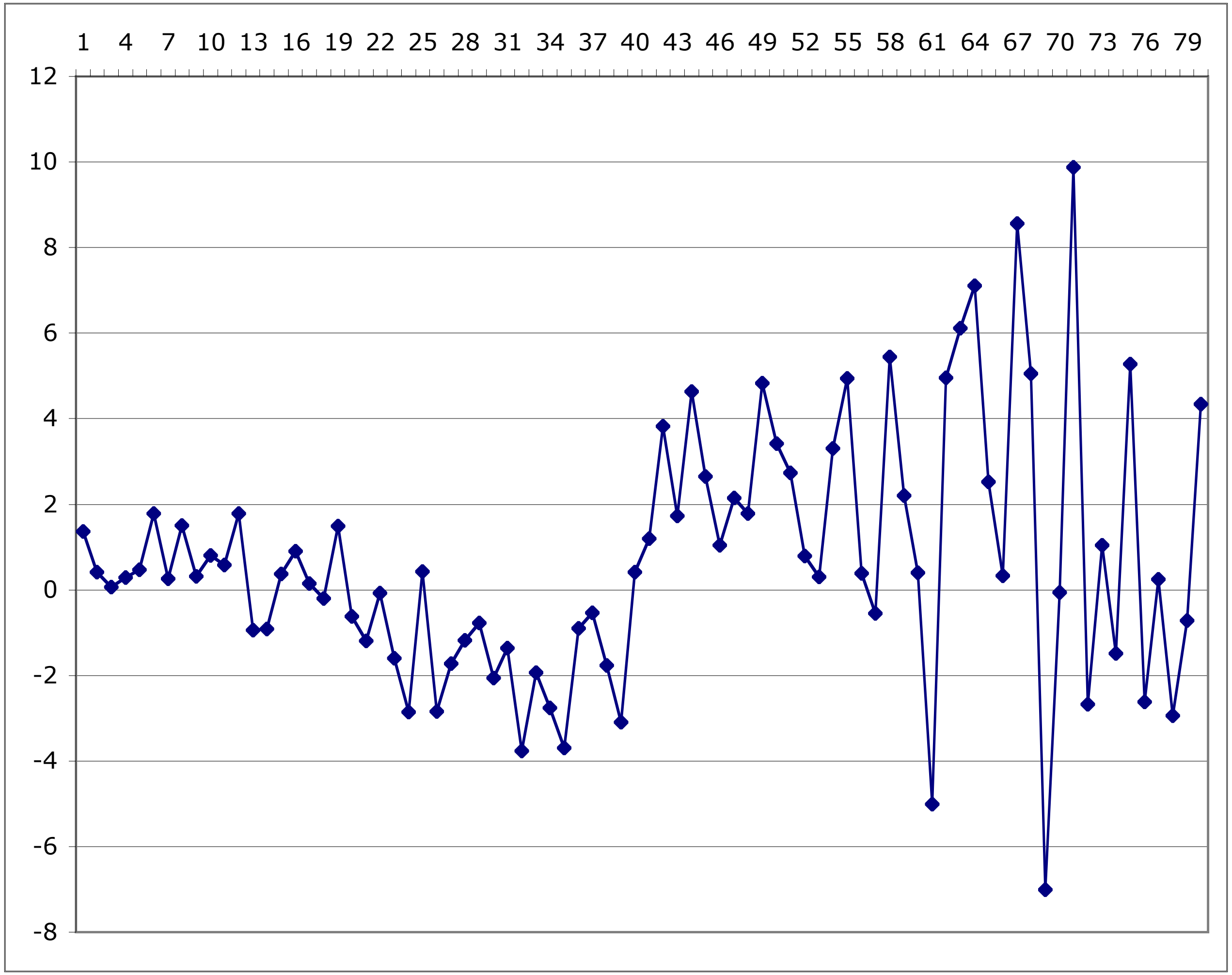
In-Control (Almost)



Not In-Control



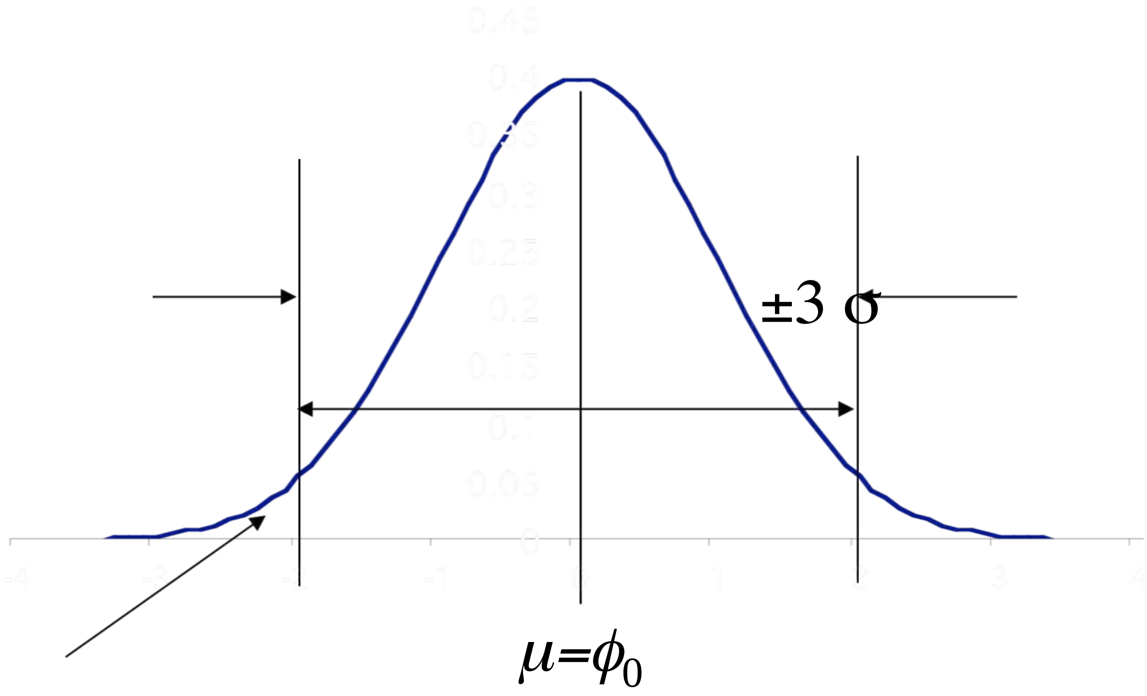
“Not In-Control”



Applying in Real-Time: Xbar and S Charts

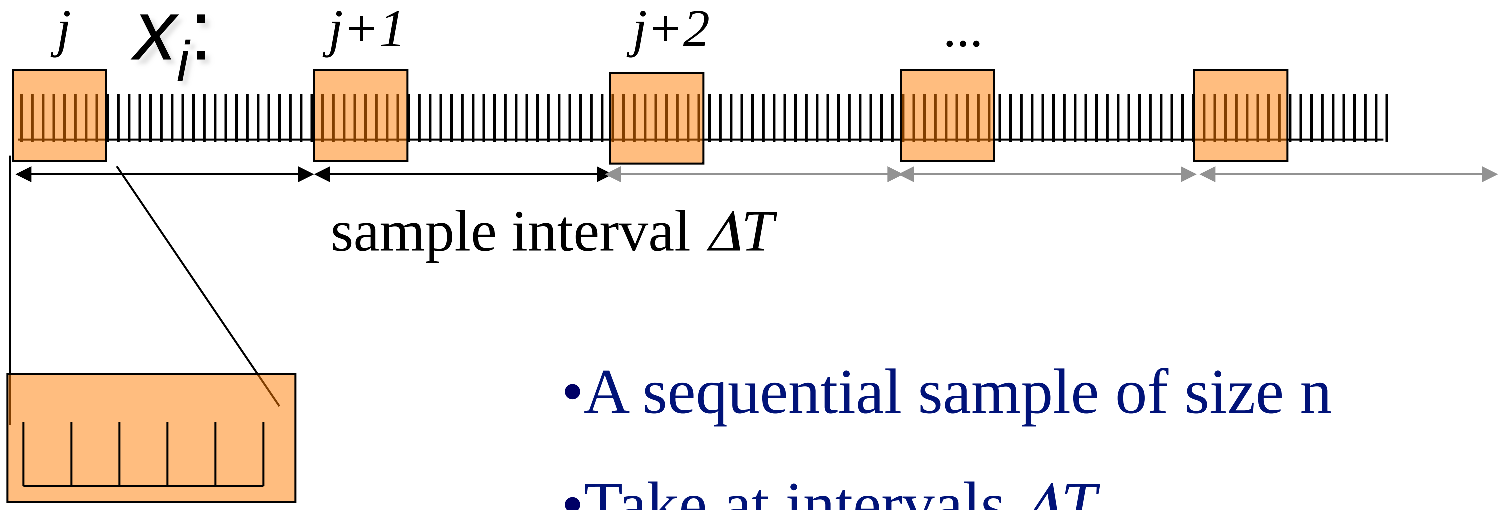
- Shewhart: Plot the Evolving Sample Statistics ($\overline{x}$ & *s*)
  - These are the estimated μ & σ for the "Normal" process model
  - Plot *sequential* ***average*** of process
    - Xbar chart
    - Distribution?
  - Plot sequential sample standard deviation
    - S chart

Process Model



Data Sampling and Sequential Averages

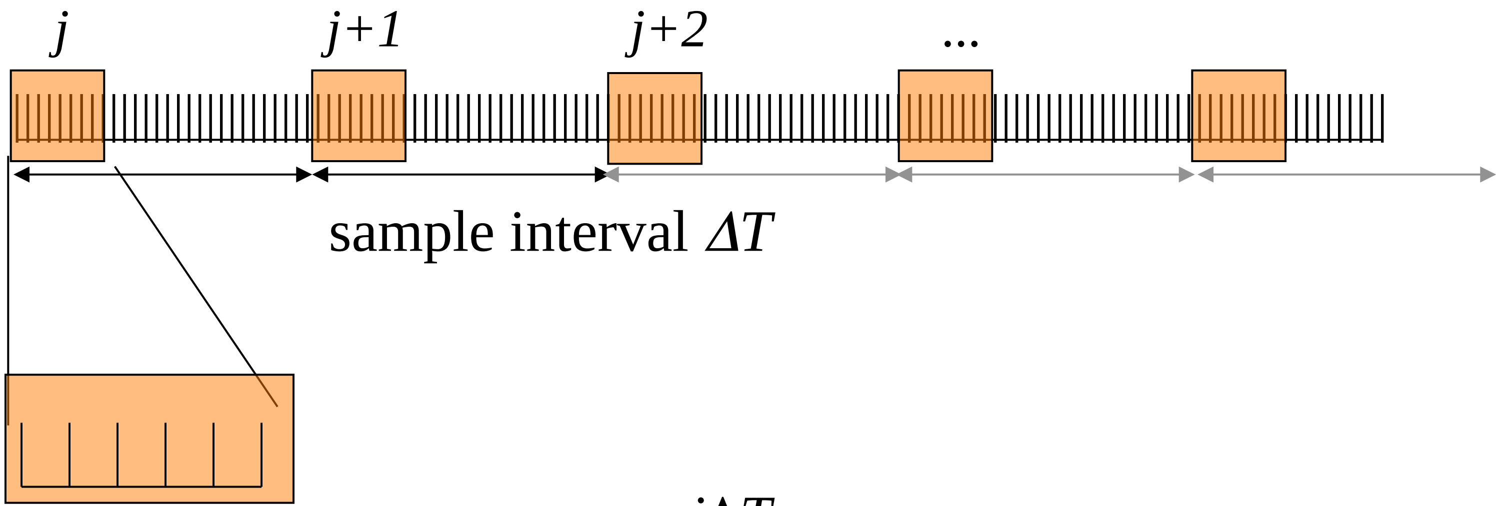
- Given a sequence of process outputs



n measurements
at sample *j*

- A sequential sample of size n
- Take at intervals *ΔT*
- Sample index j

Data Sampling



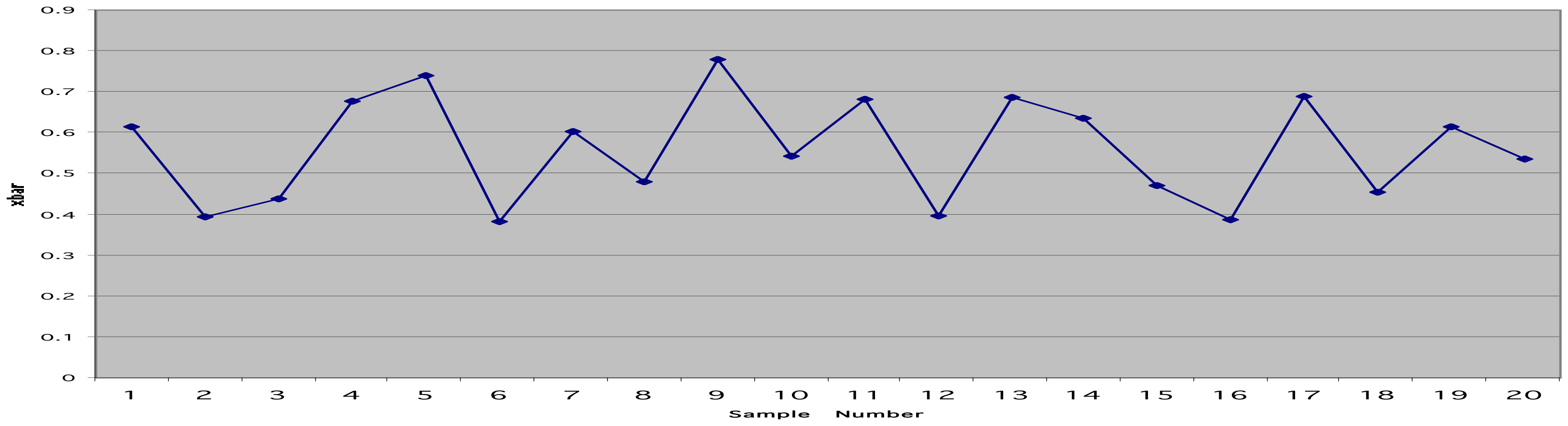
n measurements at sample *j*

$$\bar{x}_j = \frac{1}{n}\sum_{i=(j-1)\Delta T+1}^{j\Delta T+n} x_i \qquad \text{sample } j \text{ mean}$$

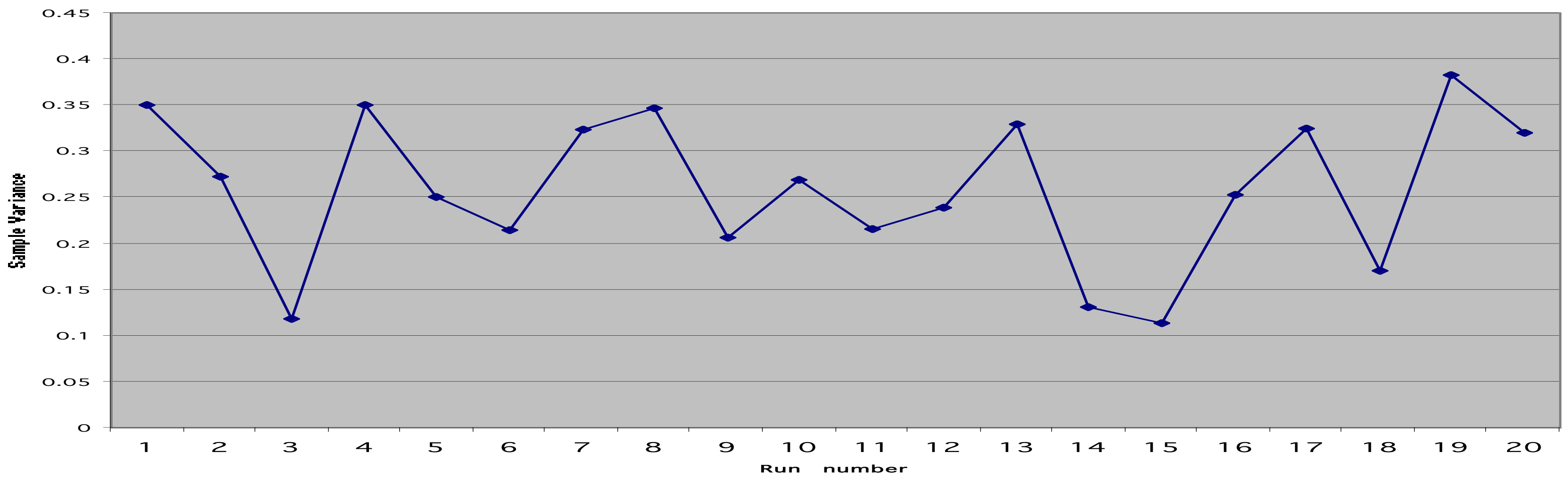
$$S_j^{\,2} = \frac{1}{n-1}\sum_{i=(j-1)\Delta T}^{j\Delta T+n} (x_i - \bar{x}_j)^2 \quad \text{sample } j \text{ variance}$$

•Plot of xbar and S
Random Data **n=5**

xbar

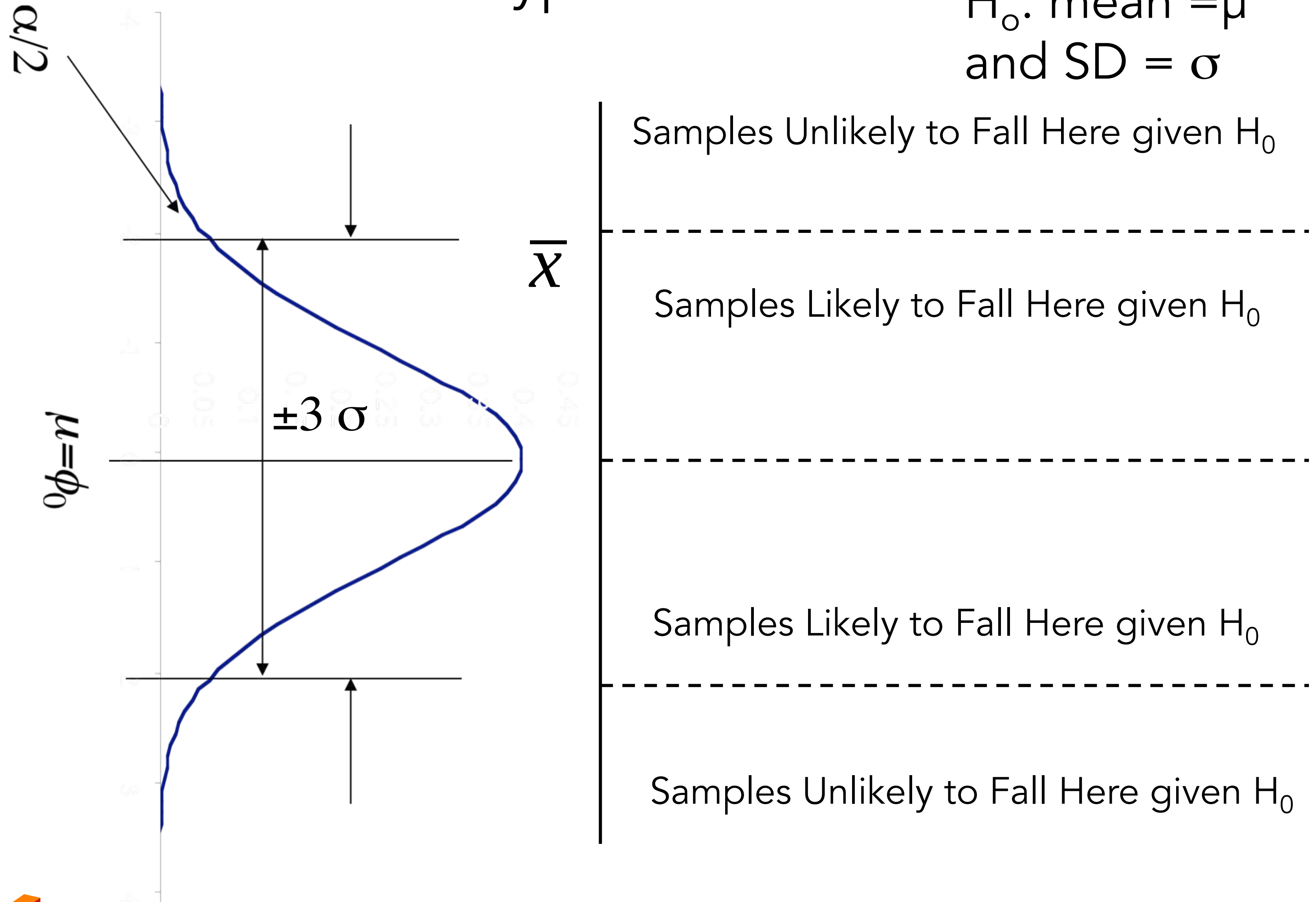


S

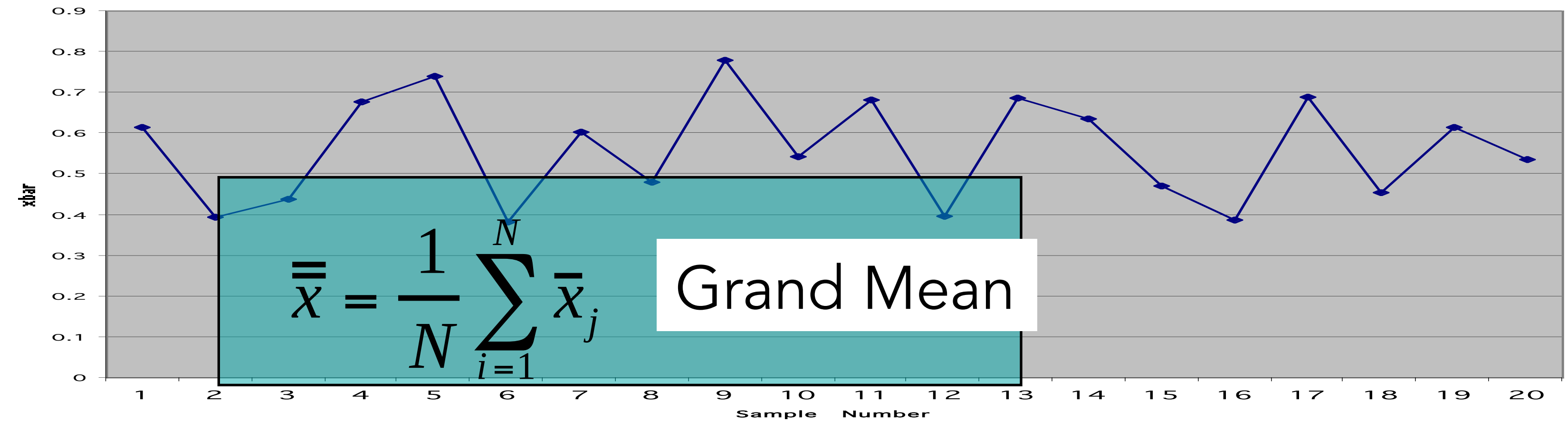


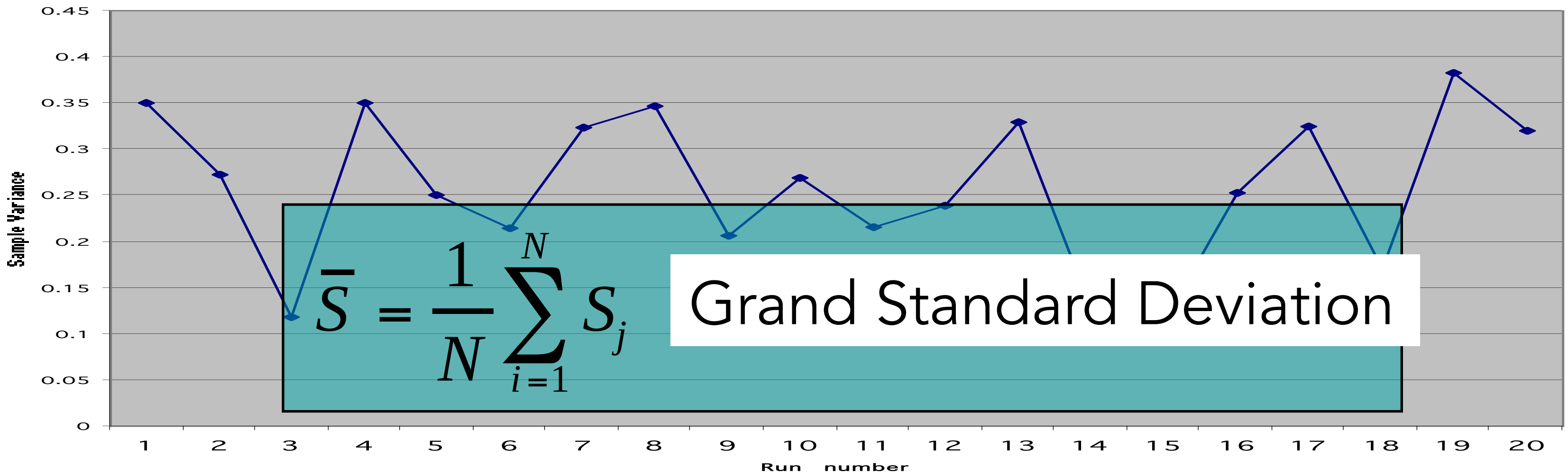
xBar Chart as Hypothesis Test

Hypothesis
H_o: mean $=\mu$
and SD = σ



Overall Statistics





Setting Chart Limits

- Expected Ranges
- Confidence Intervals
  - Intervals of $\pm$ n Standard Deviations
  - Most Typical is $\pm\ 3\sigma$

Superposition of Random Variables

If we define a variable

$$y = c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 + \ldots$$

- c_i are constants
- x_i are independent random variables

Then $$\mu_y = c_1\mu_1 + c_2\mu_2 + c_3\mu_3 + c_4\mu_4 + \ldots$$

$$\sigma_y^2 = c_1^2\sigma_\iota^2 + c_2^2\sigma_2^2 + c_3^2\sigma_3^2 + c_4^2\sigma_4^2$$

For example $y = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ $\quad c_j = \frac{1}{n}$ $\quad \sigma_{\bar{x}}^2 = \frac{1}{n}\sigma_x$

Chart Limits - Xbar

- If we knew σ_x then by superposition:

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{n}}\sigma_x$$

- But Since we *Estimate* the Sample Standard Deviation, then

$$\mathrm{E}(S_j) = C_4\sigma_{\bar{x}}$$ (S_j is a biased estimator)

$$\text{where} \quad C_4 = \left(\frac{2}{n-1}\right)^{1/2} \frac{\Gamma(n/2)}{\Gamma((n-1)/2)}$$

Chart Limits xbar chart

The estimate of *True* Sample Mean Variance (variance of the mean) is biased

To remove this bias for the xbar ± 3σ limits we use:

$$UCL = \bar{\bar{x}} + 3\frac{\bar{S}}{C_4\sqrt{n}} \qquad LCL = \bar{\bar{x}} - 3\frac{\bar{S}}{C_4\sqrt{n}}$$

For the example *n*=5 $\quad C_4 = (0.5)^{1/2}\frac{\Gamma(2.5)}{\Gamma(2)} = 0.707\frac{1.33}{1} = 0.94$

Chart Limits S

The variance of the estimate of S can be shown to be:

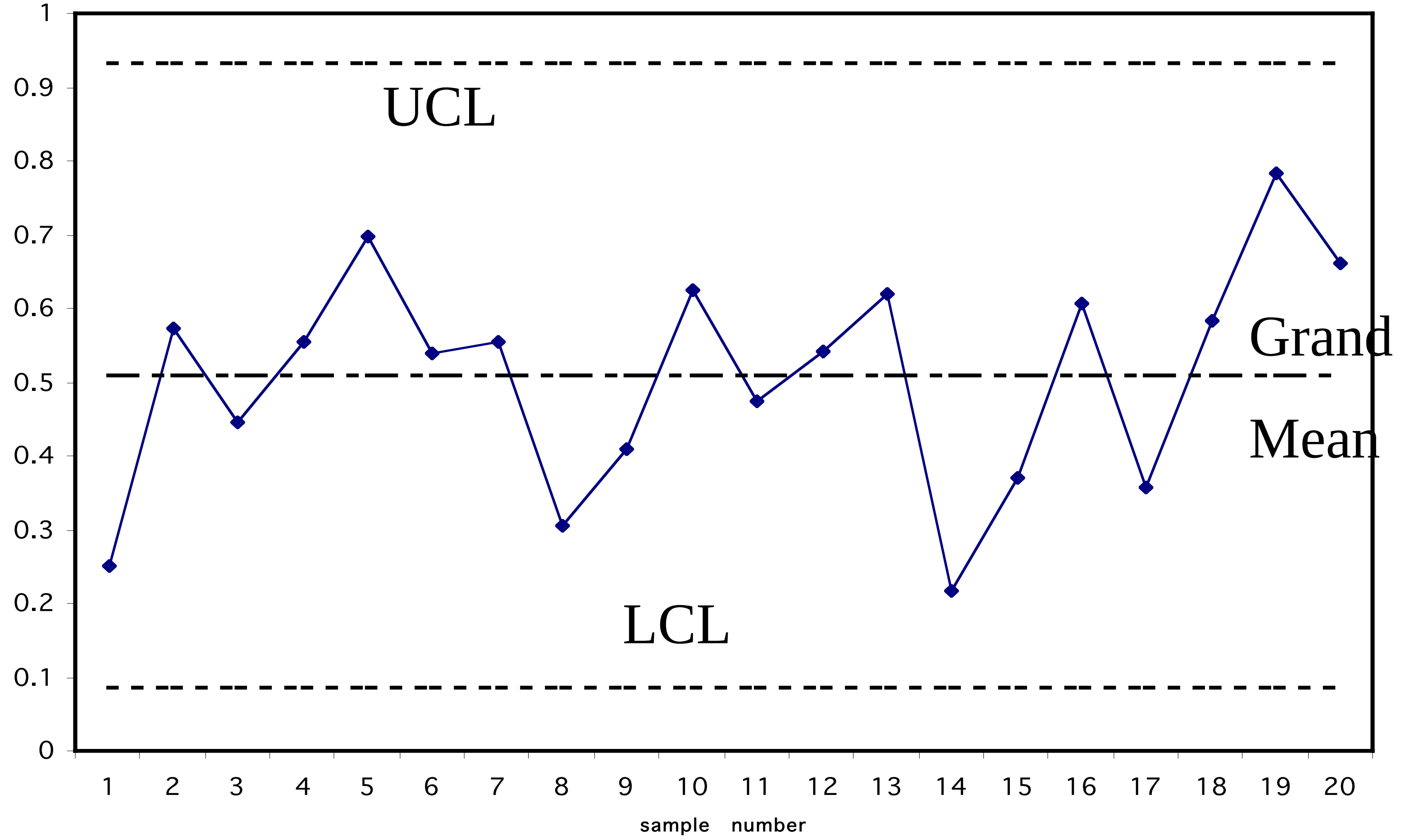
$$\sigma_S = \sigma\sqrt{1 - C_4^{\ 2}}$$

So we get the chart limits:

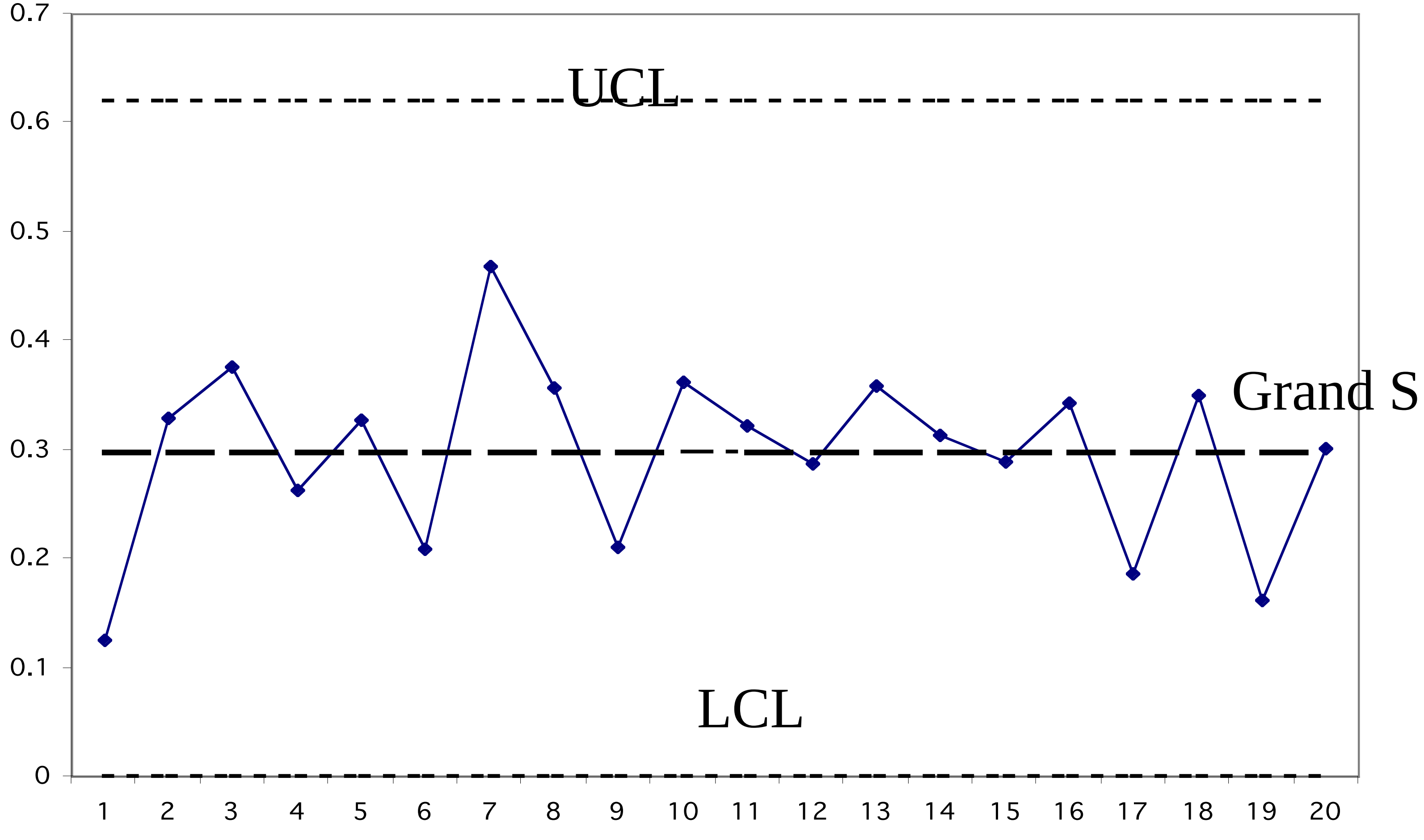
$$UCL = \bar{S} + 3\frac{\bar{S}}{C_4}\sqrt{1 - C_4^{\ 2}}$$

$$LCL = \bar{S} - 3\frac{\bar{S}}{C_4}\sqrt{1 - C_4^{\ 2}}$$

Example xbar



Example S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11
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20
UCL
Grand S
LCL

Detecting Problems from Running Data

- Appearance of data
  - Confidence Intervals
  - Frequency of extremes
  - Trends

Western Electric Rules

- Points outside limits
- 2-3 consecutive points outside 2 sigma
- Four of five consecutive points beyond 1 sigma
- Run of 8 consecutive points on one side of center

Test for "Out of Control"

- Extreme Points
  - Outside $\pm 3\sigma$
- Improbable Points
  - 2 of 3 $> \pm 2\sigma$
  - 4 of 5 $> \pm 1\sigma$
  - All points inside $\pm 1\sigma$

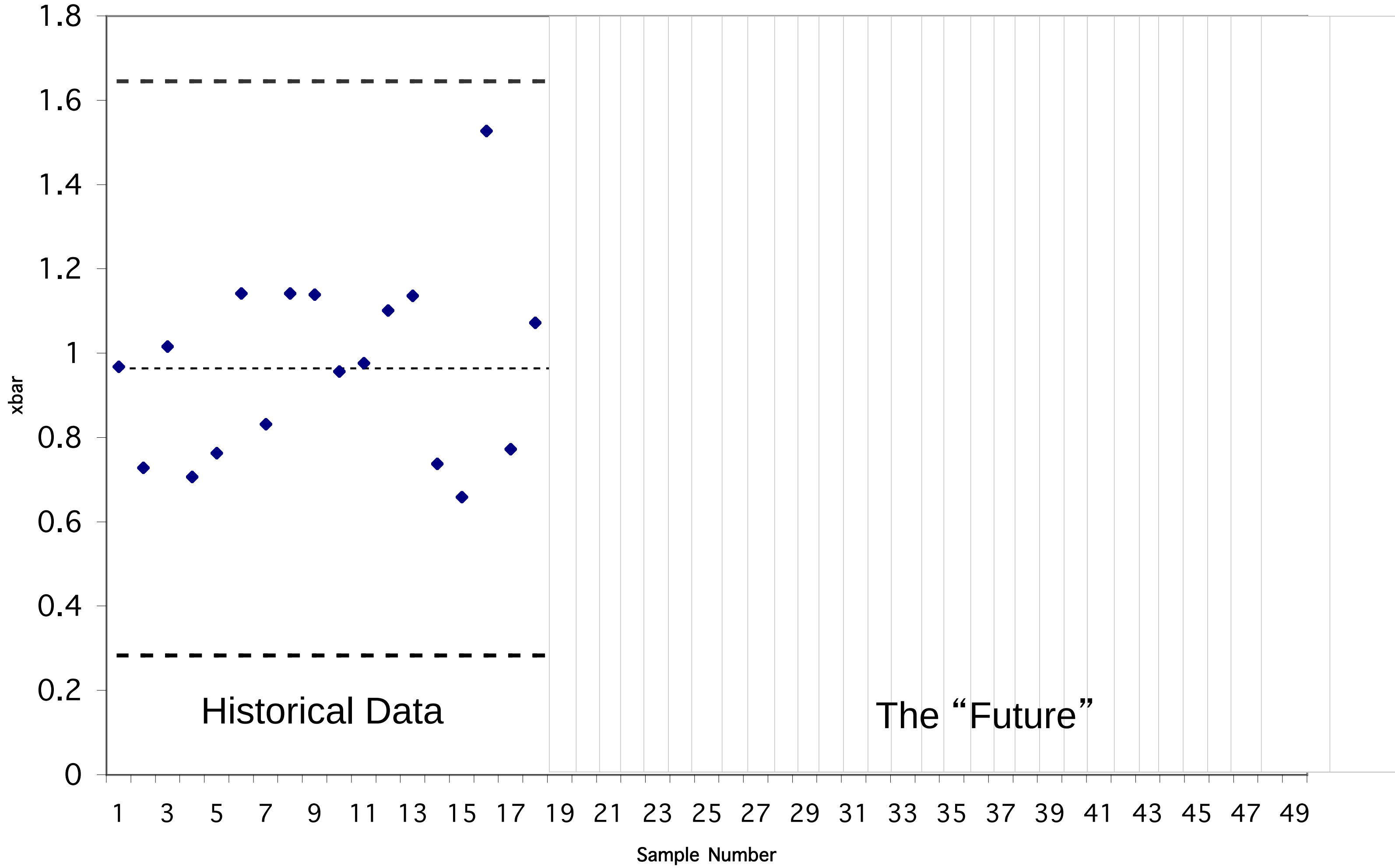
Tests for "Out of Control"

- Consistently above or below centerline
  - Runs of 8 or more
- Linear Trends
  - 6 or more points in consistent direction
- Bi-Modal Data
  - 8 successive points outside $\pm 1\sigma$

Applying Shewhart Charting

- Find a run of 25-50 points that are "in-control"
- Compute chart centerlines and limits
- Begin Plotting subsequent $xbar_j$ and S_j
- Apply rules, or look for trends, improbable events or extremes.
- If these occur, process is "out of control"

Real-Time



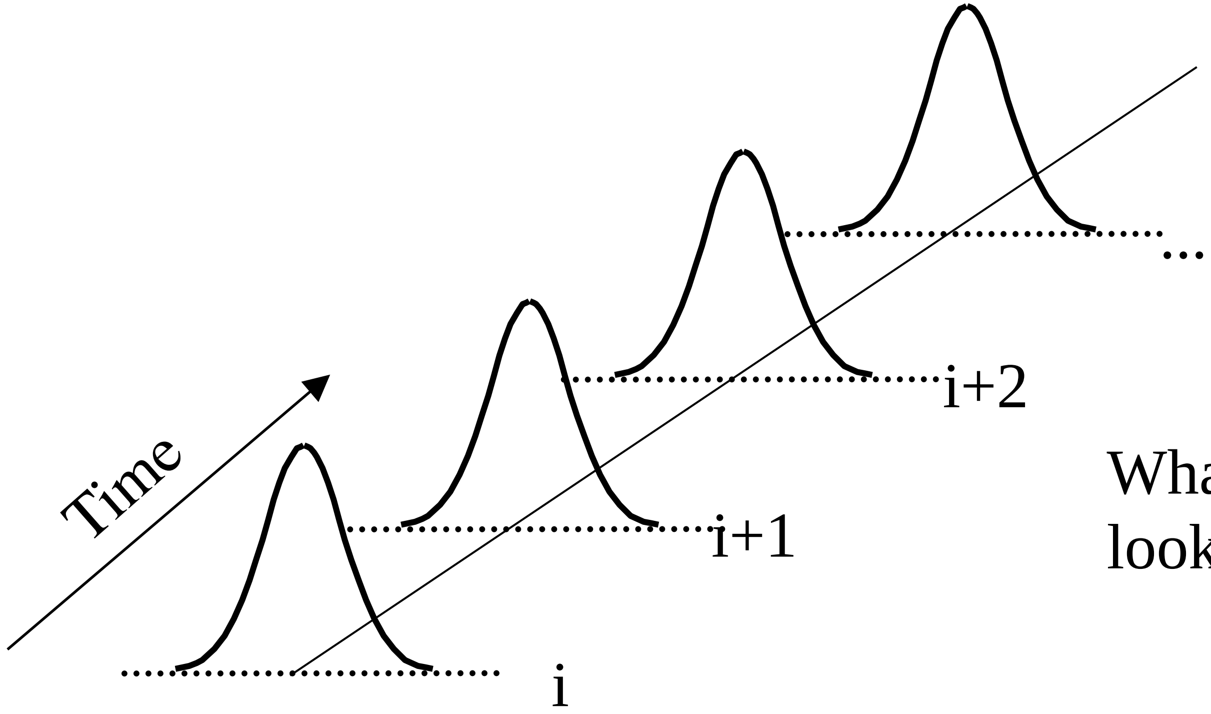
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
xbar
Historical Data
The "Future"
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49
Sample Number

Out of Control

- Data is not Stationary

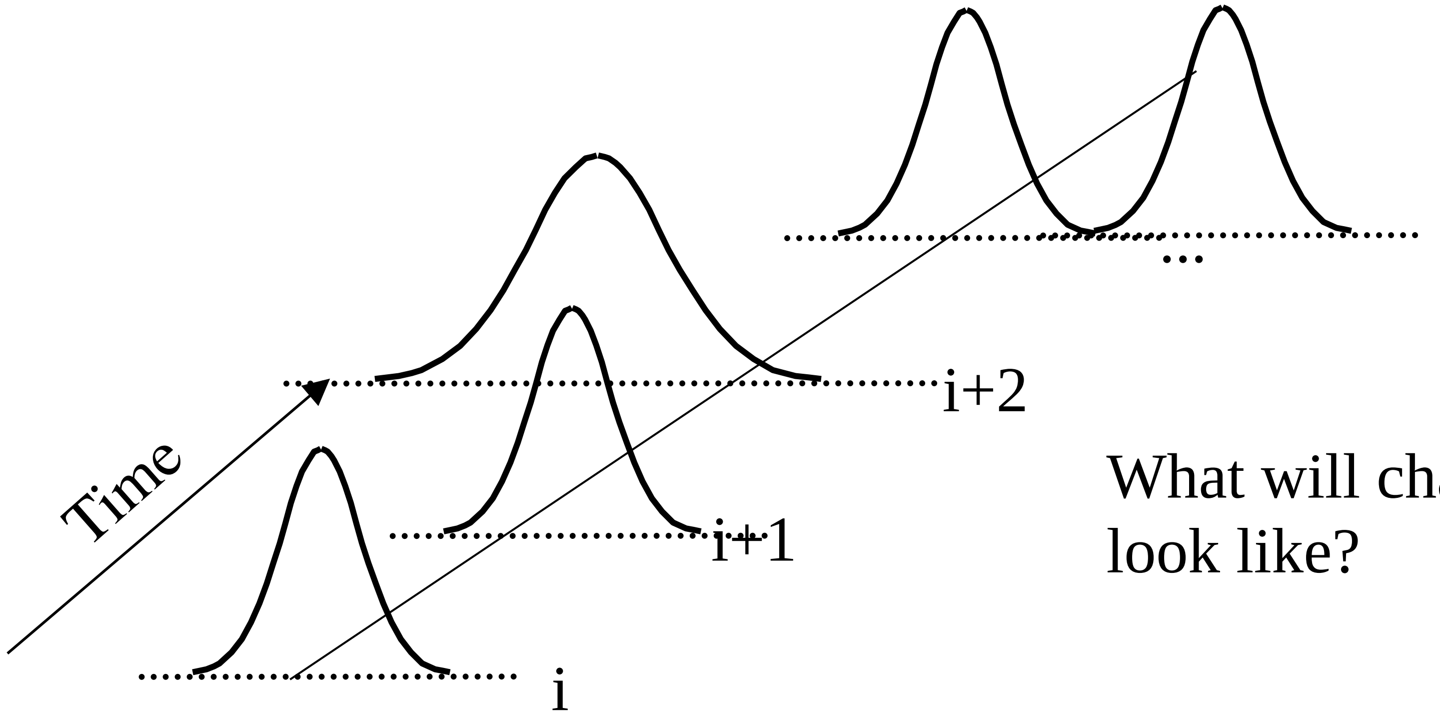
  (μ or σ are not constant)
- Process Output is being "caused" by a disturbance (common cause)
- This disturbance can be identified and eliminated
  – Trends indicate certain types
  – Correlation with know events
    - shift changes
    - material changes

“In-Control”



What will chart look like?

"Not In-Control"

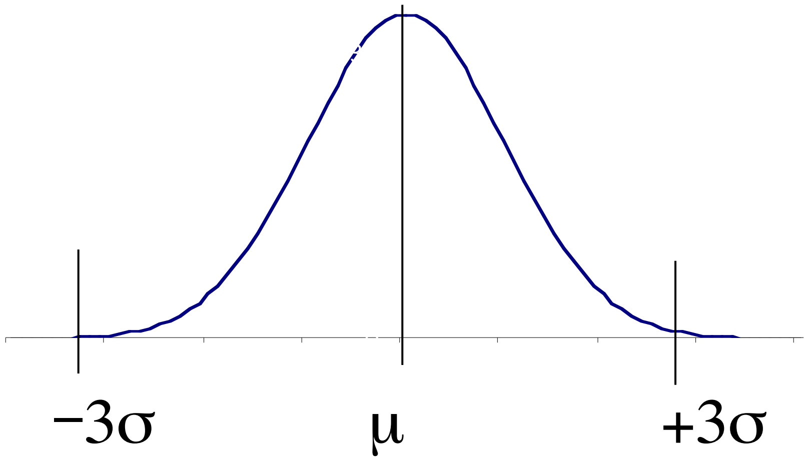


What will chart look like?

Another Use of the Statistical Process Model:

The Manufacturing -Design Interface

- We now have an empirical model of the process



How "good" is the process?

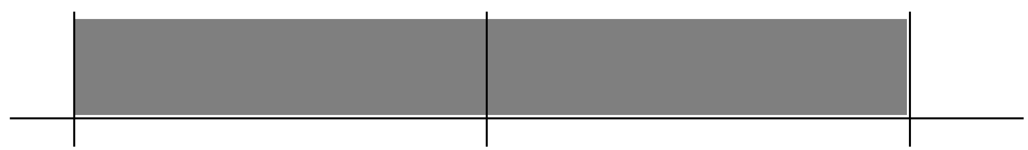
Is it capable of producing what we need?

Process Capability

- Assume Process is In-control
- Described fully by *xbar* and *s*
- Compare to Design Specifications
  - Tolerances
  - Quality Loss

Design Specifications

- **Tolerances**: Upper and Lower Limits



Characteristic Dimension

Lower Specification Limit

LSL

Target

x*

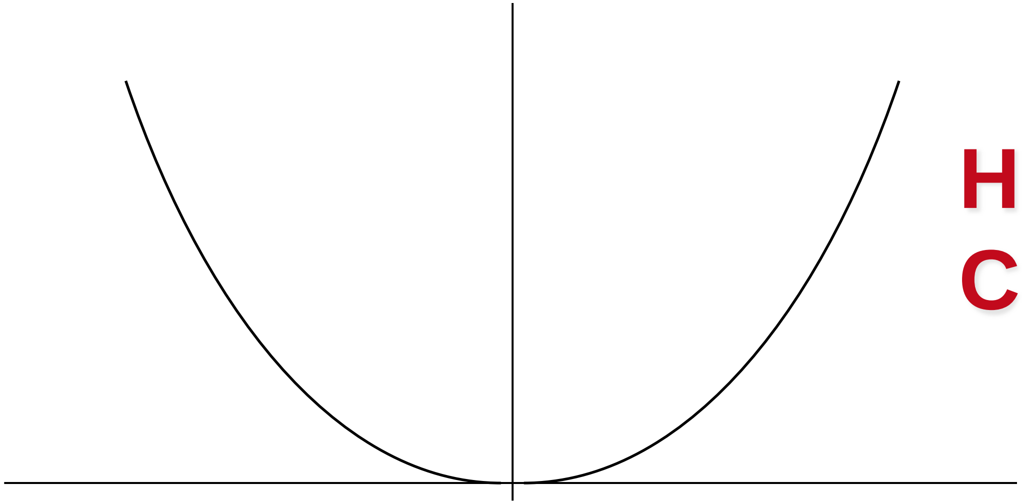
Upper Specification Limit

USL

Design Specifications

- **Quality Loss**: Penalty for Any Deviation from Target

$$QLF = L^*(x-x^*)^2$$

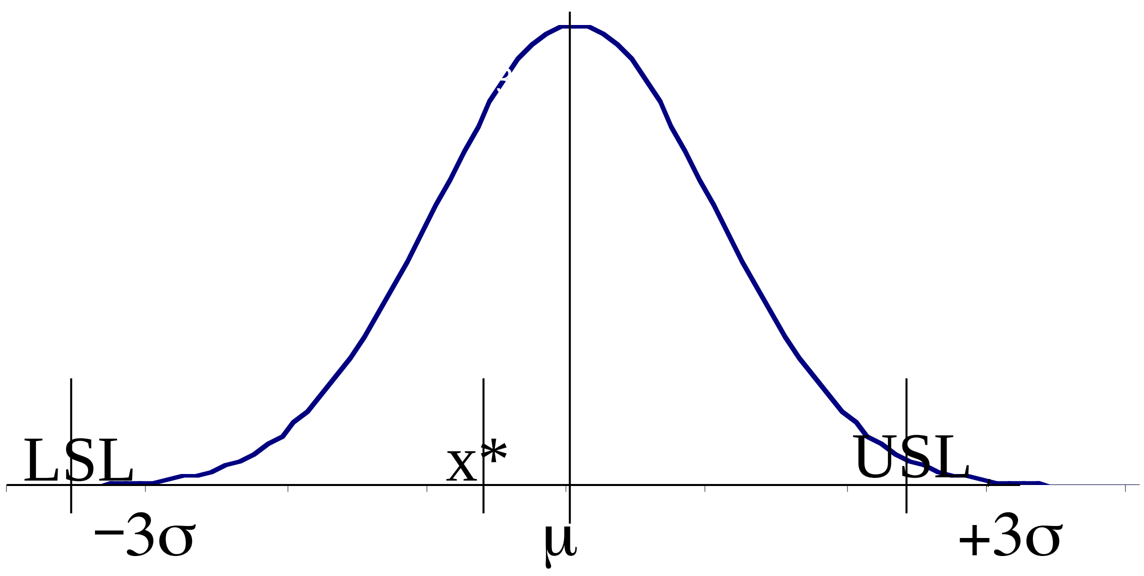


How to Calibrate?

x=target*

Use of Tolerances: Process Capability

- Define Process using a Normal Distribution
- Superimpose x*, LSL and USL
- Evaluate Expected Performance



Process Capability

- Definitions

$$C_p = \frac{(USL - LSL)}{6\sigma} = \frac{\text{tolerance range}}{\text{99.97\% confidence range}}$$

- Compares ranges only
- No effect of a mean shift:

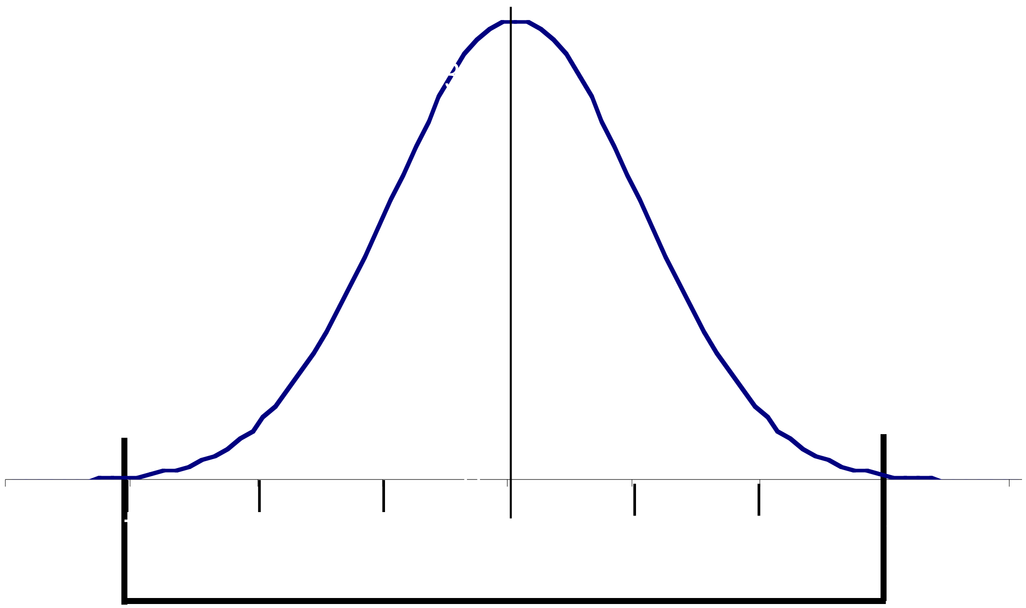
Process Capability: C_{pk}

$$C_{pk} = \min\left(\frac{(USL - \mu)}{3\sigma}, \frac{(LSL - \mu)}{3\sigma}\right)$$

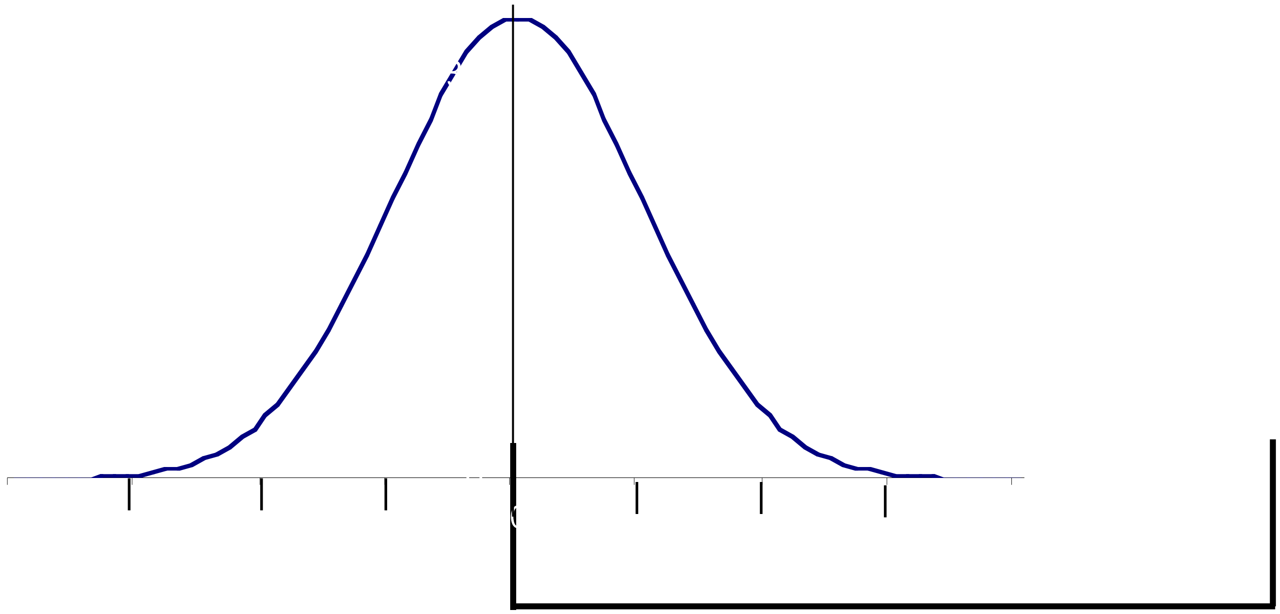
= Minimum of the normalized deviation from the mean

- Compares effect of offsets

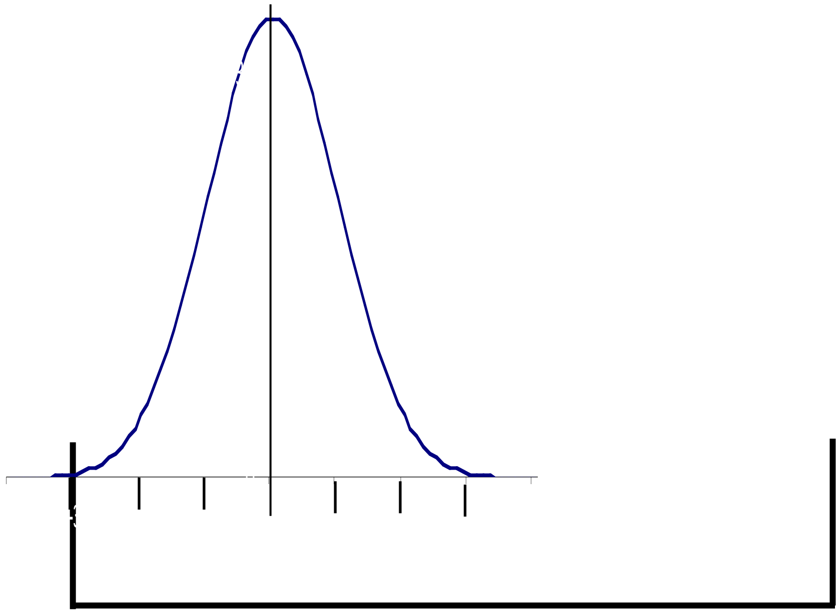
Cp = 1; Cpk = 1



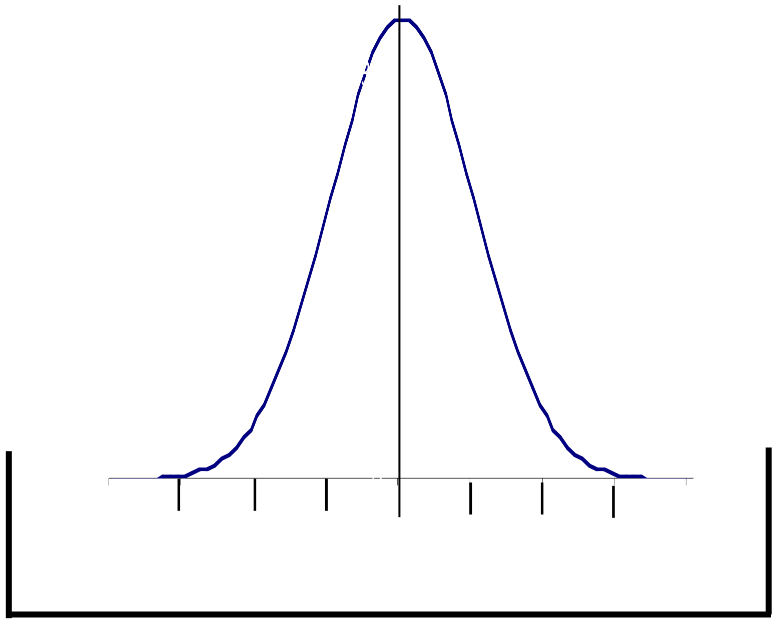
Cp = 1; Cpk = 0



Cp = 2; Cpk = 1



Cp = 2; Cpk = 2



Effect of Changes

- In Design Specs
- In Process Mean
- In Process Variance

- What are good values of Cp and Cpk?

Cpk Table

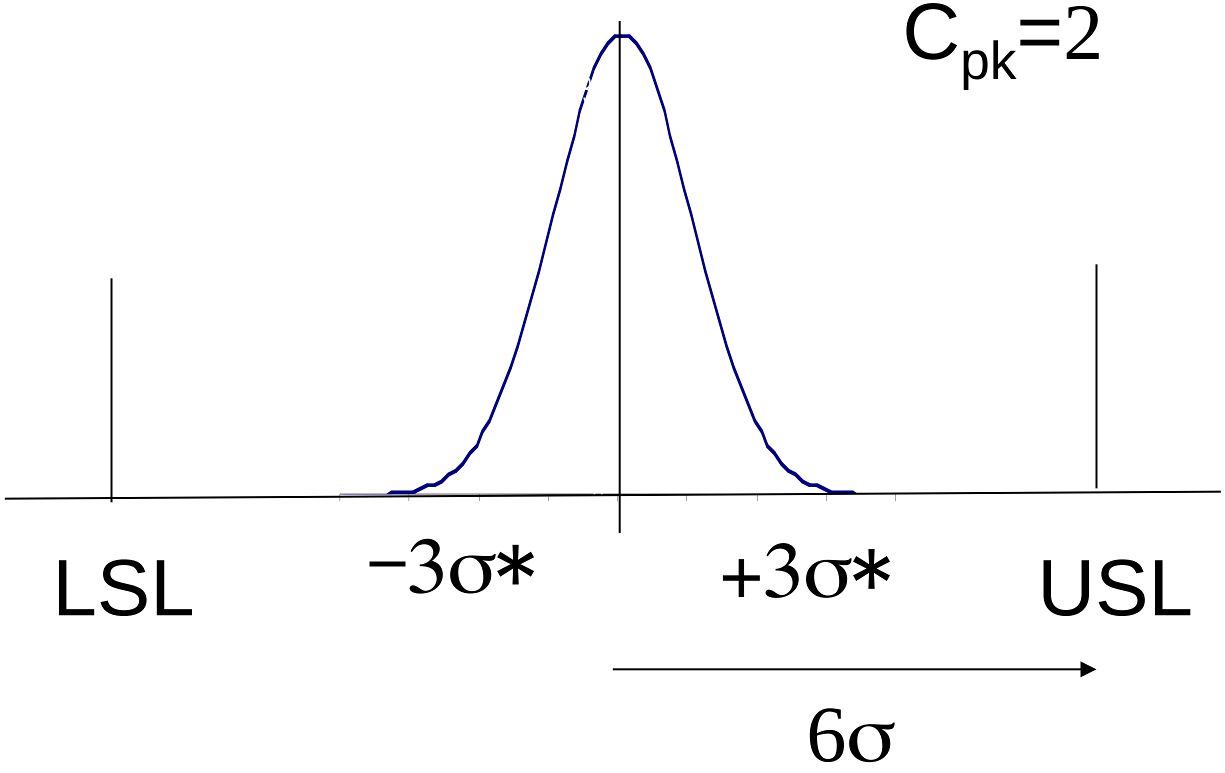
Cpk	z	P<LS or P>USL
1	3	1E-03
1.33	5	3E-07
1.67	4	3E-05
2	6	1E-09

The "6 Sigma" problem

$P(x > 6\sigma) = 18.8x10^{-10}$

$C_p=2$

$C_{pk}=2$



The 6 σ problem: Mean Shifts

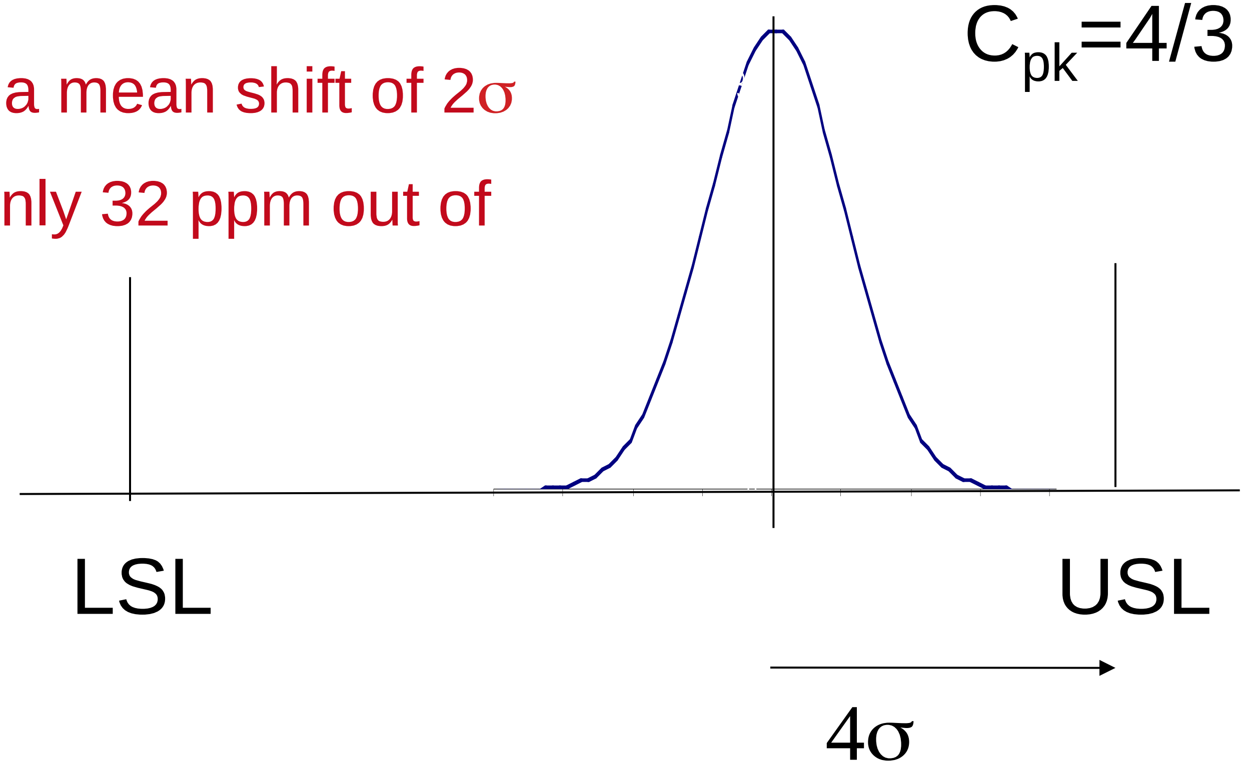
$P(x>4\sigma) = 31.6\text{x}10^{-6}$

$C_p=2$

$C_{pk}=4/3$

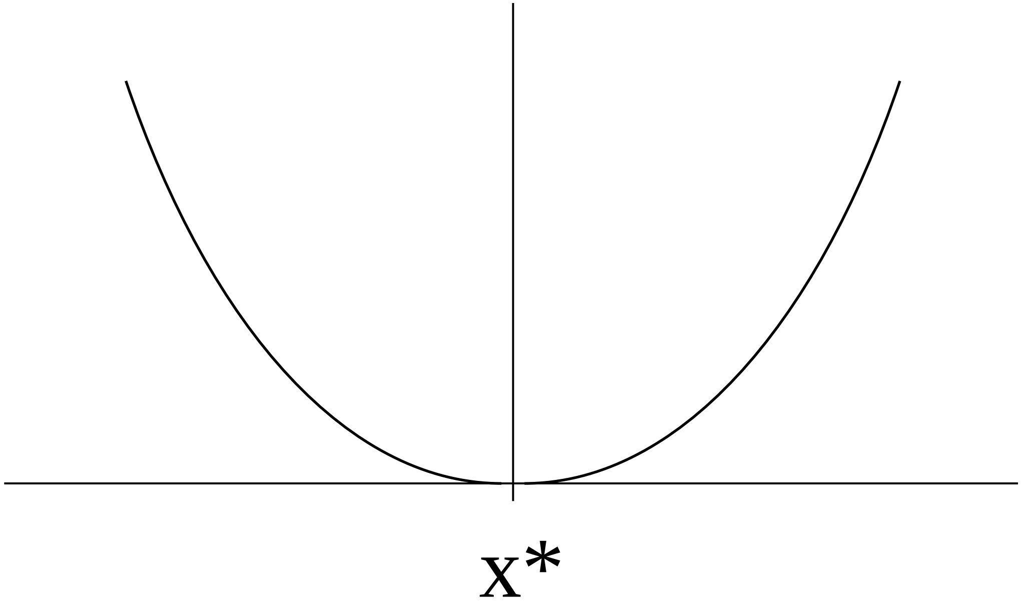
Even with a mean shift of 2σ

we have only 32 ppm out of spec



Capability from the Quality Loss Function

$$QLF = L(x) = k^*(x-x^*)^2$$



Given L(x) and p(x) what is E{L(x)}?

Expected Quality Loss

$$E\{L(x)\} = E\left[k(x - x^*)^2\right]$$

$$= k\left[E(x^2) - 2E(xx^*) + E(x^{*2})\right]$$

$$= k\sigma_x^2 + k(\mu_x - x^*)^2$$

Penalizes Variation

Penalizes Deviation

Process Capability

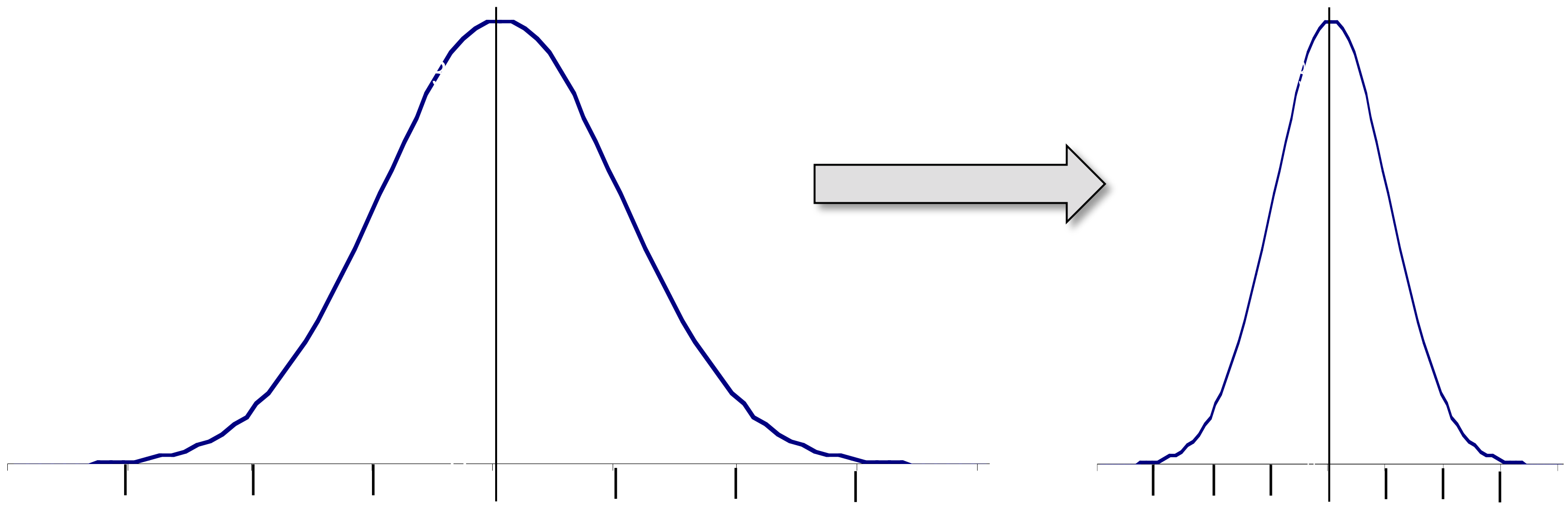
- The reality (the process statistics)
- The requirements (the design specs)
- Cp - a measure of variance vs. tolerance
- Cpk a measure of variance from target
- Expected Loss- An overall measure of goodness

Process Control Hierarchy

- Identify and Reduce Causal Disturbances
  - Good Housekeeping
  - *Standard Operations (SOP's)*
  - *Feedback Control of Machines*
    - ***Eliminate Equipment Variations***
  - Statistical Analysis and Identification of Sources (SPC)
    - ***Eliminate Assignable Causes***

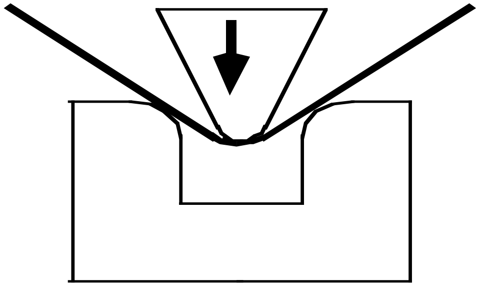
Process Control Hierarchy

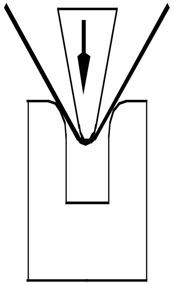
- NEXT: Reduce *Sensitivity* to Disturbance
  - Measure Sensitivities via Designed Experiments (DOE)
  - Adjust "free" parameters to minimize variations



Example: Bending Sensitivity to Yield Stress

Simple Example: Die Width for Air Bending (An adjustable equipment property):

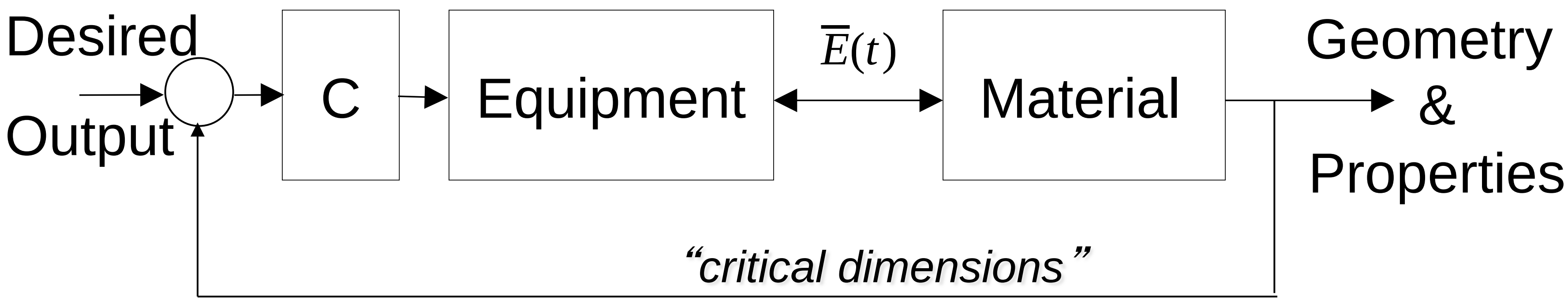




- Wide Die:
  - Low force,
  - high spring back,
  - high sensitivity to variations in yield stress

- Narrow Die:
  - High force,
  - Higher material stress,
  - Lower spring back,
  - Lower sensitivity to variations in yield stress,

Final Step :"Output Control"



Examples:

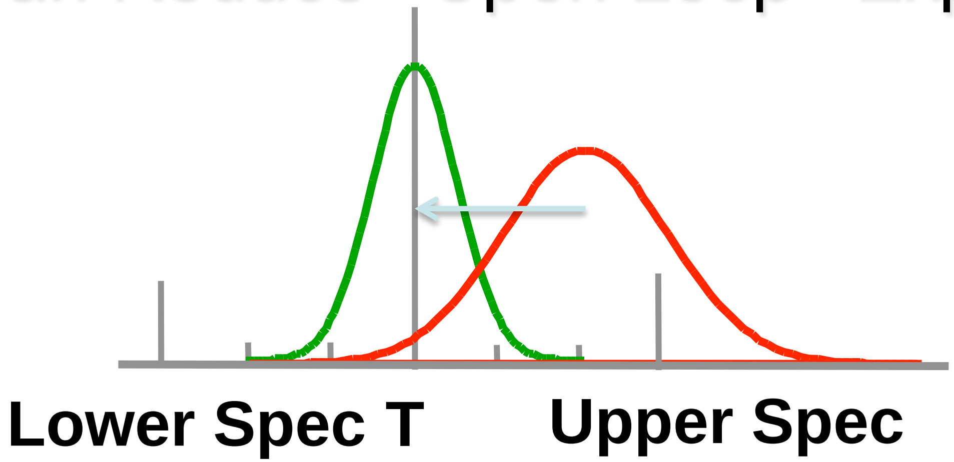
- Web Thickness in Milling
- Sheet Thickness in Rolling
- Sheet Angle in Bending

Implementing Product Feedback Control

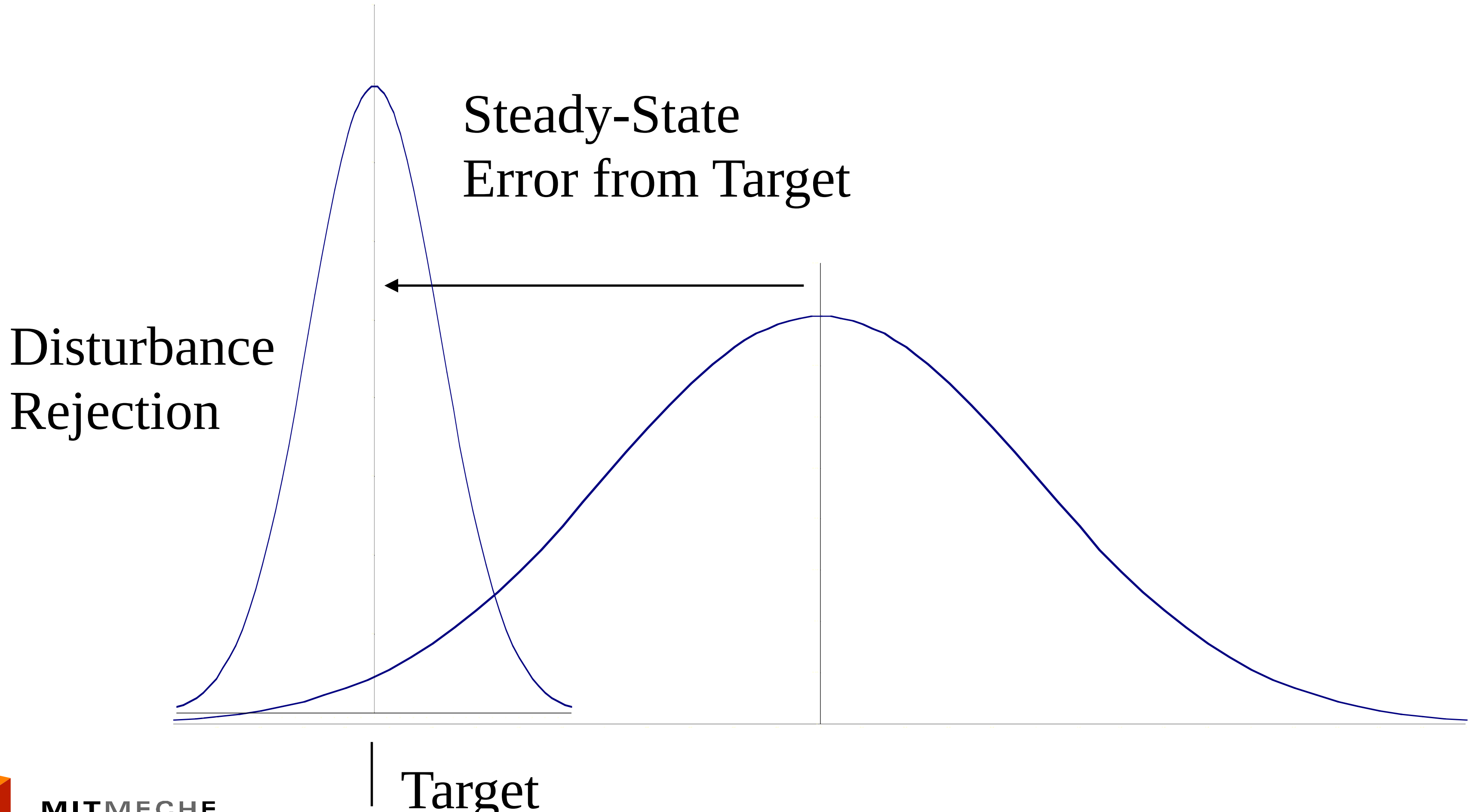
- Continuous In-Process Measurements
  - Regulate Process States In-Process
- Sampling and Monitoring (SPC)
  - Measure After-Process and Diagnose
- **Part to Part Sampling and Control**
  - Cycle to Cycle Control: Measure After each Cycle and Improve Process Capability

Conclusions: Single Variable Case

- Cycle to Cycle Control
  - Obeys Root Locus Prediction wrt Dynamics
  - Amplifies White Noise Disturbance
    Attenuates Colored Noise Disturbance
  - Can Reduce Mean Error (Zero if I-control)
  - Can Reduce "Open Loop" Expected Loss

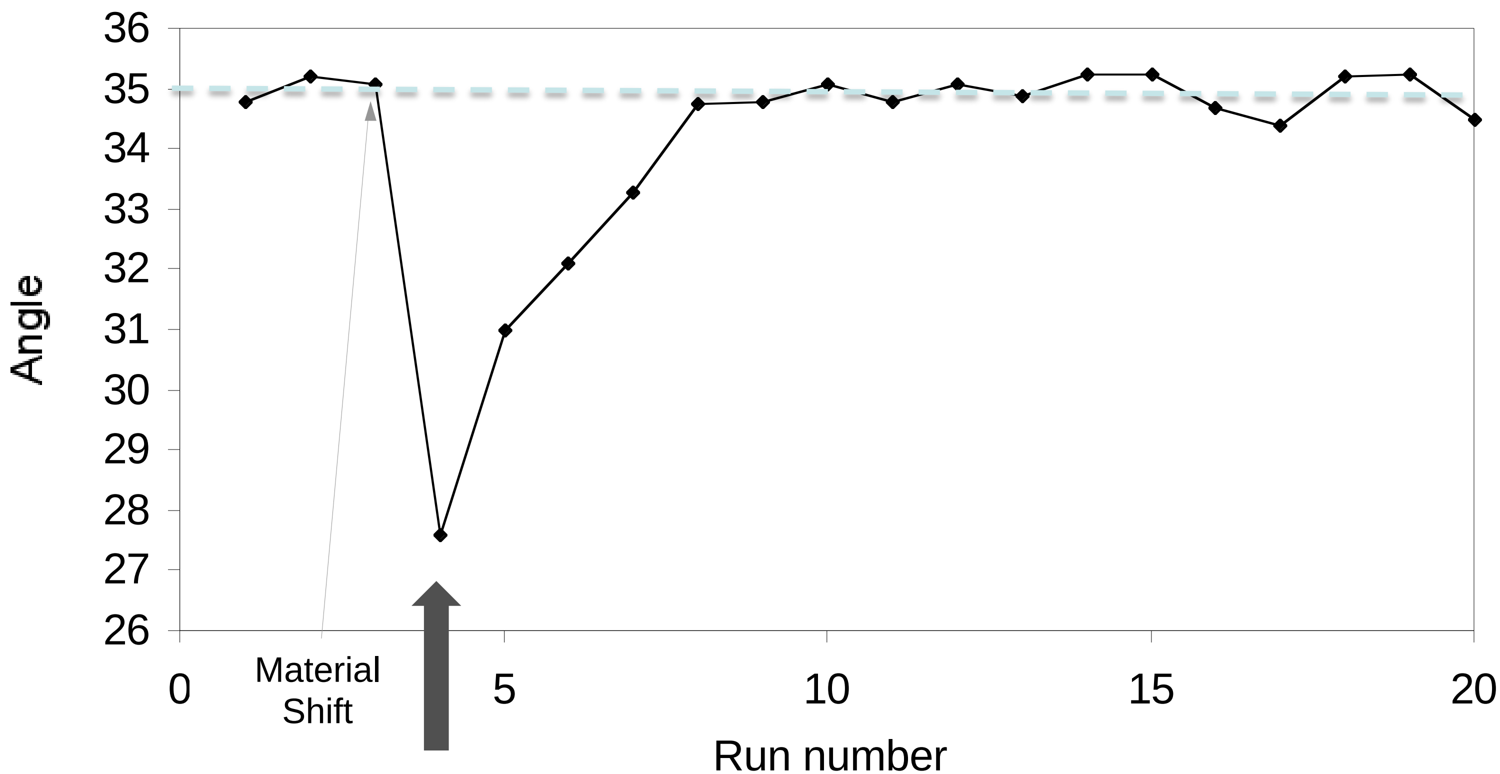


Feedback Control Objectives

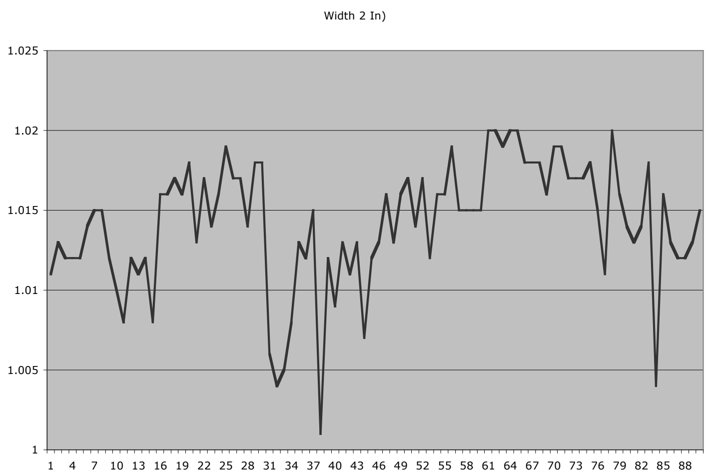


It Works!: Bending Step Disturbance

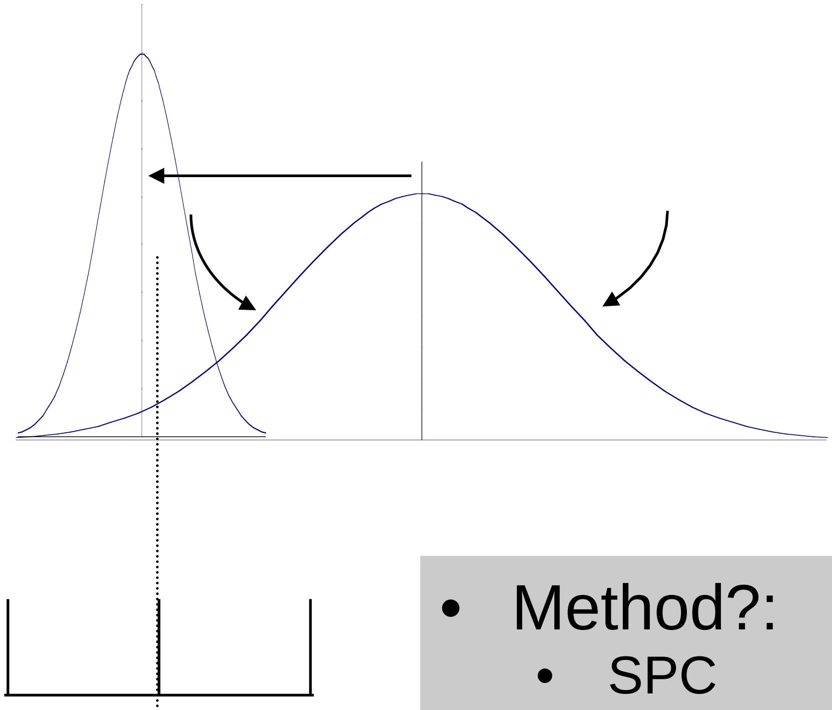
- Effect of Material Change
  - Switch to a Stiffer Material – more springback.



Manufacturing Objective



Process Output



- Method?:
  - SPC
  - Optimization
  - Output Feedback

Conclusions

- Shewhart Charts
  - Application of Statistics to Production
  - Plot Evolution of Sample Statistics $\overline{x}$ and S
  - Look for Deviations from Model
- Process Capability
  - A measure of the process to meet a requirement
  - Includes variance and bias
  - Gets design and manufacturing talking
- If That’s Not Good Enough
  - DOE/Optimization
  - Feedback Control
  - …