Variability in Manufacturing Systems

Stanley B. Gershwin
gershwin@mit.edu
http://web.mit.edu/manuf-sys
Massachusetts Institute of Technology

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Manufacturing Systems
Definitions

- *Manufacturing:* the transformation of material into something useful and portable.

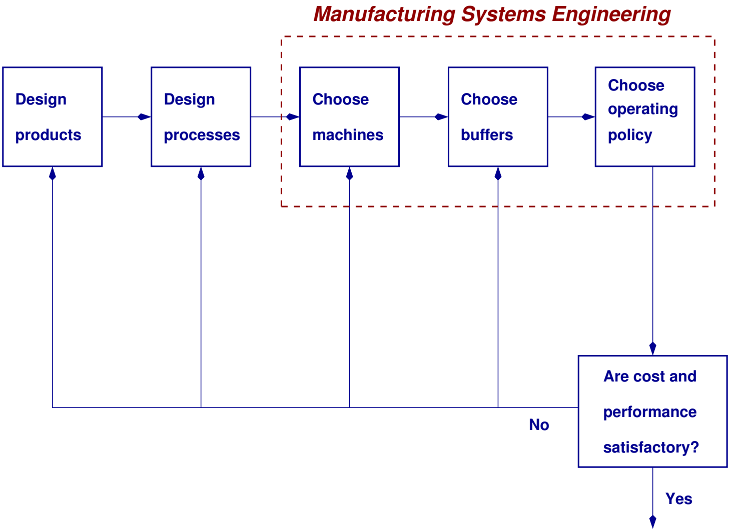
- *Manufacturing System:* A manufacturing system is a set of machines, transportation elements, computers, storage buffers, people, and other items that are used together for manufacturing.
  - Alternate terms:
    - *Factory*
    - *Production system*
    - *Fabrication facility*
  - Subsets of manufacturing systems, which are themselves systems, are sometimes called *cells,* *work centers,* or *work stations* .

Manufacturing Systems
Manufacturing Systems Engineering

- The design and operation of new manufacturing systems;

- The analysis and improvement of existing manufacturing systems.

Manufacturing Systems
Manufacturing Systems Engineering



- Short product lifetimes. Frequent factory reconfiguration or replacement. Limited time for real-time learning to optimize factory.
- Large product diversity. Factories must be flexible.
- Short lead times and impatient customers.
- Inventory is perishable. It loses value rapidly due to obsolescence.
- Design and operation of manufacturing systems must take place in the presence of variability, uncertainty, and randomness.

- Manufacturing systems must be complex to manage or tolerate variability, uncertainty, and randomness.
  - ⋆ *Variability:* change over time.
  - ⋆ *Uncertainty:* incomplete knowledge.
  - ⋆ *Randomness:* unpredictability that has some regularity that can be described precisely.

- Improvements in the design and operation of manufacturing systems must
  - ⋆ ***reduce*** the variability, uncertainty, and randomness, or
  - ⋆ reduce the ***sensitivity*** of systems to variability, uncertainty, and randomness.

Manufacturing Systems Engineering Objectives

- Satisfy demand.
- Meet due dates.
- Keep quality high.
- Keep inventory low.

- ***Be robust.***
  - ⋆ Be insensitive to disruptions.
  - ⋆ Respond gracefully to disruptions.
  - ⋆ Respond gracefully to demand changes, engineering changes, etc.

- **Design Quality** – the design of products that give customers what they want or would like *(features).*

- **Manufacturing Quality** – the manufacturing of products to *avoid* giving customers what they *don't* want or *would not* like *(defects).*

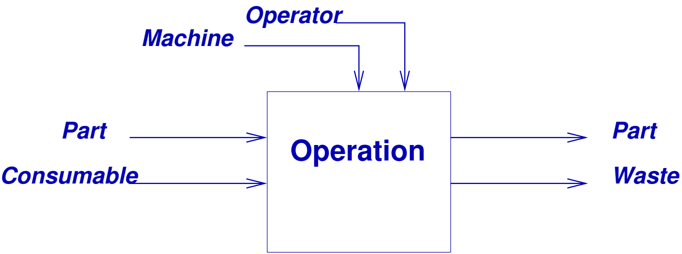
Manufacturing Systems
Quantity, Quality, and Variability

- Quantity – *how much* is produced and *when* it is produced.
- Quality – *how well* it is produced.

In this session, we focus on *quantity*.

General Statement: Variability is the enemy of manufacturing.

Why care about variability?
What is an operation?



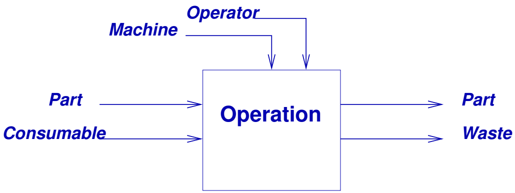
Nothing happens until everything is present.

Why care about variability?
What is an operation?

Whatever does not arrive last must wait.

- *Inventory:* parts waiting.
- *Under-utilization:* machines waiting.
- *Idle work force:* operators waiting.

Why care about variability?
What is an operation?



- *Reductions* in the availability, or ...
- *Increased variability* in the availability ...

... of any one of these items increases waiting in the rest of them and reduces the performance of the system.

Why care about variability?
Examples

- Factories are full of random events:
  - ⋆ machine failures
  - ⋆ changes in orders
  - ⋆ quality failures
  - ⋆ human variability
- The economic environment is uncertain:
  - ⋆ demand variations
  - ⋆ supplier unreliability
  - ⋆ changes in costs and prices

Why care about variability?

Therefore, factories should be

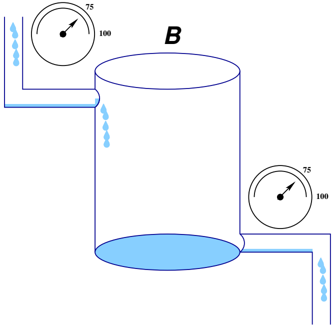
- *designed* and *operated*

to minimize the

- *creation*, *propagation*, or *amplification*

of *uncertainty*, *variability*, and *randomness*.

Variability and Inventory
No variability

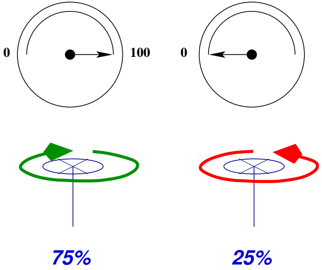


75 gal/sec in, 75 gal/sec out constantly

The tank is always empty.

Variability and Inventory
Variability from random valves

Consider a random valve:



- The average period when the valve is open is 15 minutes.
- The average period when the valve is closed is 5 minutes.
- Consequently, the *average* flow rate through it is 75 gal/sec*.

* *... as long as flow is not impeded upstream or downstream.*

Variability and Inventory
Variability from random valves

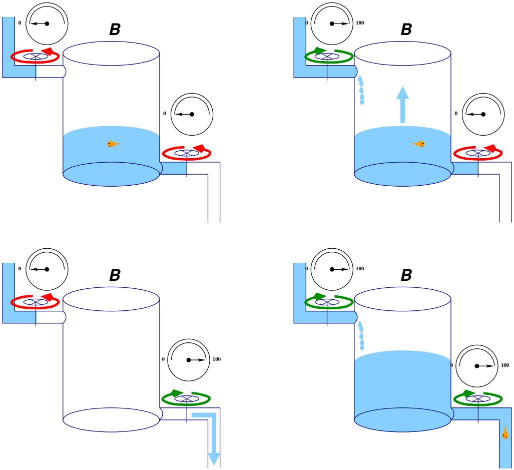
Four of the possibilities for *two* valves and one tank:

0 in, 0 out

100 gal/sec in,
0 out

0 in, 100
gal/sec out

100 gal/sec in,
100 gal/sec ou



Variability and Inventory

Observation:

- There is never any water in the tank when the flow is constant.
- There is sometimes water in the tank when the flow is variable.

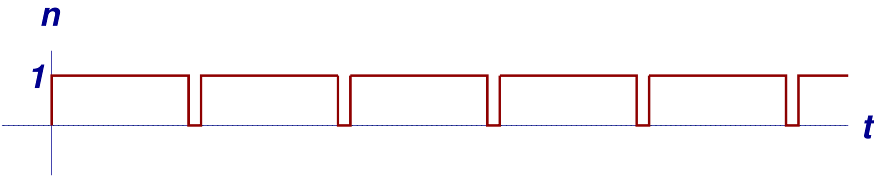
Conclusions:

1. You can't always replace random variables with their averages.
2. *Variability causes inventory!!*

Variability and Inventory

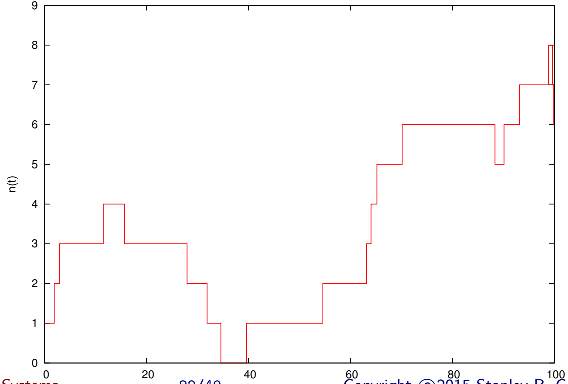
- To be more precise, *non-synchronization causes inventory.*
    - ⋆ Living things do not acquire energy at the same time they expend it. Therefore, they must store energy in the form of fat or sugar.
    - ⋆ Rivers are dammed and reservoirs are created to control the flow of water — to reduce the variability of the water supply.
    - ⋆ For solar and wind power to be successful, energy storage is required for when the sun doesn't shine and the wind doesn't blow.

- Suppose customers arrive at time 0, time 1 minute, 2 minutes, etc. and that service time is always exactly 54 seconds (.9 minutes).
  - ⋆ Then there is 1 customer in the system for 54 seconds of every minute and 0 customers for 6 seconds every minute. Therefore there are *54/60=.9* customers in the system on the average.

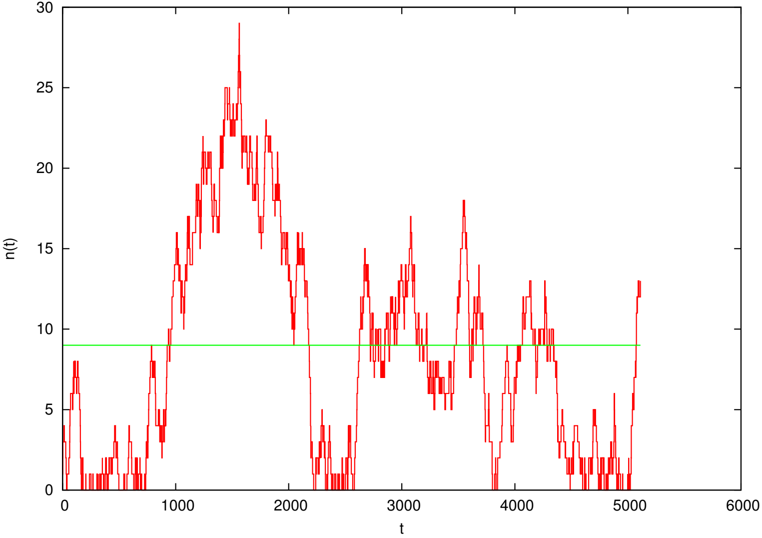


Variability and Inventory
Queues

- Suppose customers arrive with exponentially distributed inter-arrival times with *average* inter-arrival time 1 minute; and that service time is exponentially distributed with *average* service time 54 seconds.
  - ⋆ Then the average number of customers in the system is *9.*



Variability and Inventory
Queues



Let $\rho = \lambda/\mu$. Under the assumptions of exponential inter-arrival time, exponential service time, and infinite buffer (called *M/M/1 queue*), it can be shown that the average number of parts in the system is

$$\bar{n} = \sum_n nP(n) = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu - \lambda}.$$

ρ is called the *utilization* of the server.

Variability and Inventory
Queues

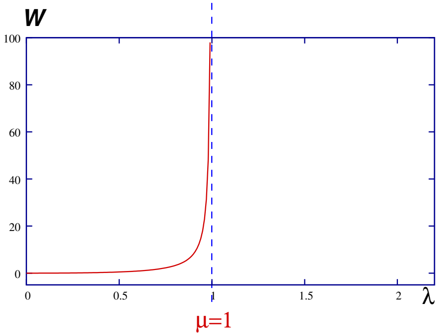
Little's Law: $L = \lambda W$, where L is the average number of customers in a system, W is the average customer waiting time, and λ is the average arrival rate.

This is true for almost all queuing systems in steady state.

In the $M/M/1$ queue, $L = \bar{n}$, so the average waiting time is

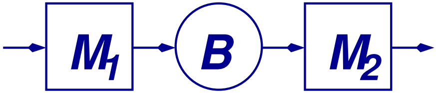
$$W = \frac{1}{\mu - \lambda}.$$

ρ is called the *utilization* of the server.



- μ is the *capacity* of the system.
- If $\lambda < \mu$, system is stable and waiting time remains bounded.
- If $\lambda > \mu$, waiting time grows over time.

Inventory and the Propagation of Variability
Two-machine production lines



- A simple production line consisting of two machines and one in-process inventory buffer, making discrete parts.
- Both machines have unit operation time.
- Both machines are unreliable with geometric uptimes and downtimes. The parameters are
  - ⋆ $\text{MTTR}_1 = 1/r_1$, $\text{MTTR}_2 = 1/r_2$
  - ⋆ $\text{MTTF}_1 = 1/p_1$, $\text{MTTF}_2 = 1/p_2$
- The buffer can hold a maximum of N parts.
- The first machine is never starved; the last machine is never blocked.

- The *efficiency* e_i of Machine i is the fraction of time the machine is operational, *not counting idle time* .

- Because the time unit is the operation time, the efficiency of a machine is the same as its production rate in isolation, i.e., not counting idle time.

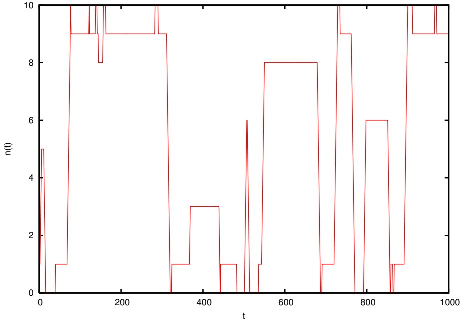
- The efficiency of Machine i is given by

$$e_i = \frac{\text{MTTF}_i}{\text{MTTR}_i + \text{MTTF}_i} = \frac{\text{MTTF}_i}{\text{MTBF}_i} = \frac{r_i}{r_i + p_i}$$

Simulations: Buffer Level vs. t

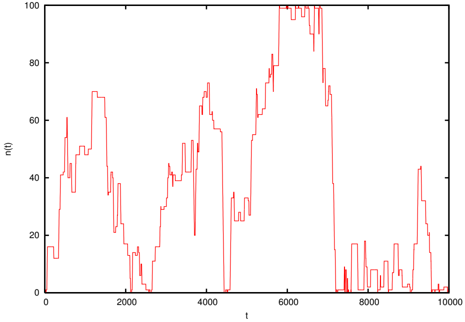
- The next four slides show simulation results. They show how the buffer level ($n(t)$) varies with time t as a result of random failures and repairs of the two machines.
- Compare the slides in order to relate the buffer level behavior to the machine parameters.
- Think about the average buffer level.

Simulations: Buffer Level vs. t



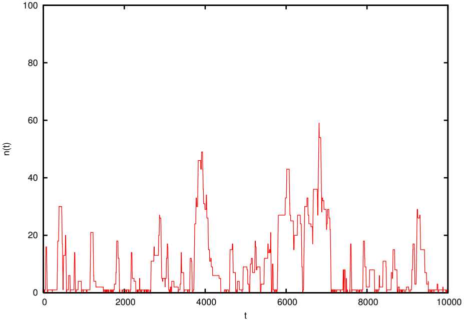
$$\text{MTTR}_i = 10, \text{MTTF}_i = 100, i = 1, 2; N = 10$$
$$(r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01)$$

Simulations: Buffer Level vs. t



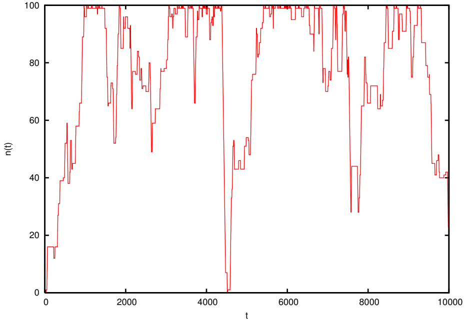
$$\text{MTTR}_i = 10, \text{MTTF}_i = 100, i = 1, 2; N = 100$$
$$(r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01)$$

Simulations: Buffer Level vs. t



$$\text{MTTR}_i = 10, i = 1, 2, \text{MTTF}_1 = 50, \text{MTTF}_i = 100; N = 100$$
$$(r_1 = .1, p_1 = .02, r_2 = .1, p_2 = .01)$$

Simulations: Buffer Level vs. t



$$\text{MTTR}_i = 10, i = 1, 2, \text{MTTF}_1 = 100, \text{MTTF}_i = 50; N = 100$$
$$(r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .02)$$

Inventory and the Propagation of Variability
Two-machine production lines

Scenario:

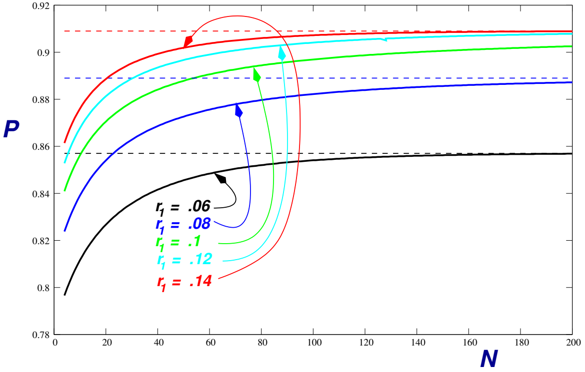
- We are designing a line. We have already chosen the second machine.
- Two decisions are left: the choice of the first machine and the size of the buffer,
- Five vendors produce versions of the first machine.
  - ⋆ All have the same operation time as the second machine.
  - ⋆ All have the same MTTF.
  - ⋆ They differ in MTTR. The vendors put different amounts of design effort into making the machines easy to repair.

Inventory and the Propagation of Variability

Two-machine production lines

Production rate vs. Buffer Size

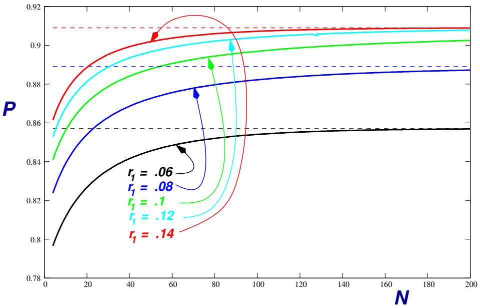
$p_1 = .01$
$r_2 = .1$
$p_2 = .01$



Inventory and the Propagation of Variability
Two-machine production lines

Discussion:

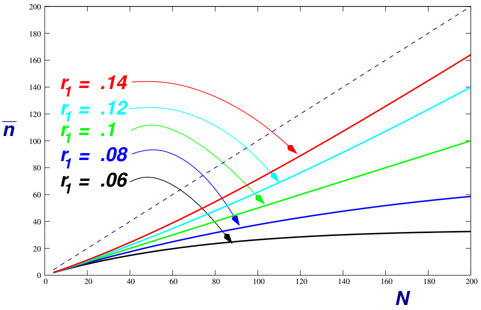
- What is the limit of P as $N \to \infty$?
- Why are the curves increasing?
- Why do they reach an asymptote?
- Why are the curves with smaller r_1 lower?



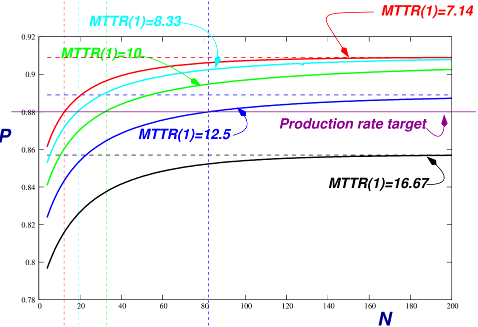
Average Inventory vs. Buffer Size

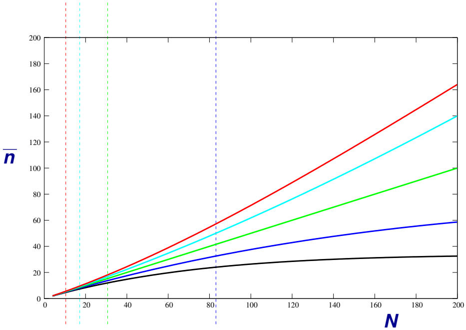
Discussion:

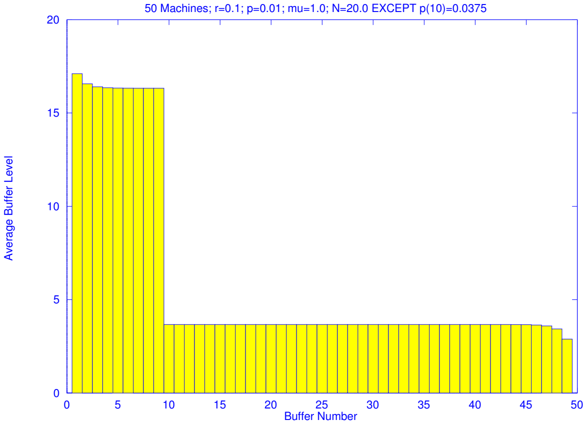
- Why are the curves increasing?
- Why *different* asymptotes?
- What is $\bar{n}$ when $N = 0$?
- What is the limit of $\bar{n}$ as $N \to \infty$?
- Why are the curves with smaller r_1 lower?



Inventory and the Propagation of Variability
Two-machine production lines







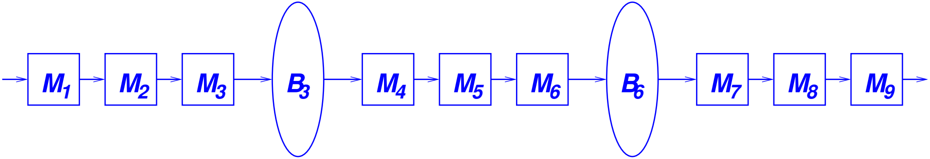
Effect of a bottleneck. Identical machines and buffers, except for M_{10}.

Inventory and the Propagation of Variability
Long production lines

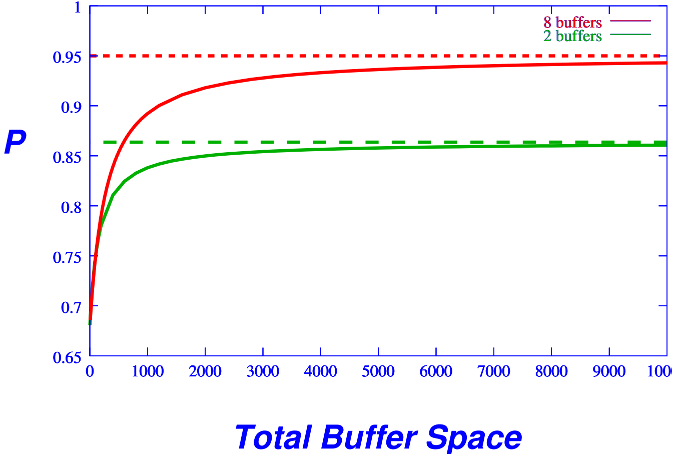
Which has a higher production rate?

- 9-Machine line with two buffering options:
  - ⋆ 8 buffers equally sized; and

  - ⋆ 2 buffers equally sized.



Inventory and the Propagation of Variability
Long production lines



Optimal buffer space distribution

- Design the buffers for a 20-machine production line.
- The machines have been selected, and the only decision remaining is the amount of space to allocate for in-process inventory.
- *The goal is to determine the smallest amount of in-process inventory space so that the line meets a production rate target.*

- The common operation time is one operation per minute.
- The target production rate is .88 parts per minute.

- *Case 1* MTTF= 200 minutes and MTTR = 10.5 minutes for all machines ($P = .95$ parts per minute).

- *Case 1* MTTF= 200 minutes and MTTR = 10.5 minutes for all machines ($P = .95$ parts per minute).
- *Case 2* Like Case 1 except Machine 5. For Machine 5, MTTF = 100 and MTTR = 10.5 minutes ($P = .905$ parts per minute).

- *Case 1* MTTF$=$ 200 minutes and MTTR $=$ 10.5 minutes for all machines ($P = .95$ parts per minute).
- *Case 2* Like Case 1 except Machine 5. For Machine 5, MTTF $=$ 100 and MTTR $=$ 10.5 minutes ($P = .905$ parts per minute).
- *Case 3* Like Case 1 except Machine 5. For Machine 5, MTTF $=$ 200 and MTTR $=$ 21 minutes ($P = .905$ parts per minute).

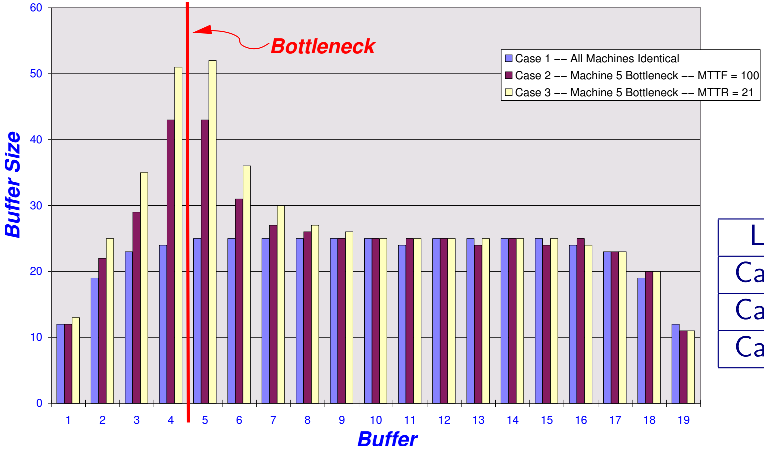
Are buffers really needed?

Line	Production rate with no buffers, parts per minute
Case 1	.487
Case 2	.475
Case 3	.475

Yes. *These numbers came from a zero-buffer formula.*

Inventory and the Propagation of Variability
Long production lines

Solution



Line	Space
Case 1	430
Case 2	485
Case 3	523

Inventory and the Propagation of Variability
Long production lines

- Observation from studying buffer space allocation problems:
  - ⋆ *Buffer space is needed most where buffer level variability is greatest!*

Other issues

- Setup changes
  - ⋆ Controllable disruption
  - ⋆ Reduces production time and creates inventory.
- Deterministic scheduling and MRP
  - ⋆ Recalculation required when an unanticipated event occurs
  - ⋆ This can cause instability and confusion

