Lecture 8

Impedance

Roadmap

In the last lecture we showed how we can represent a voltage waveform that changes in time another way – as a sum of tones (sinewaves). This representation contains the same information, just stored in a different way. These two representations are called transforms of each other.

This lecture will show you why this alternative representation is so useful. If you can represent an input as a sum of sinewaves, then it is very easy to compute the output of a circuit with resistors and capacitors. To show how this is done, we will introduce the concept of **impedance**, which is a generalization of resistance. Using impedance we can then estimate how the gain of circuit can change with frequency, creating **filters**, and how the response of these filters are graphed in a **Bode plot**.

Learning Objectives

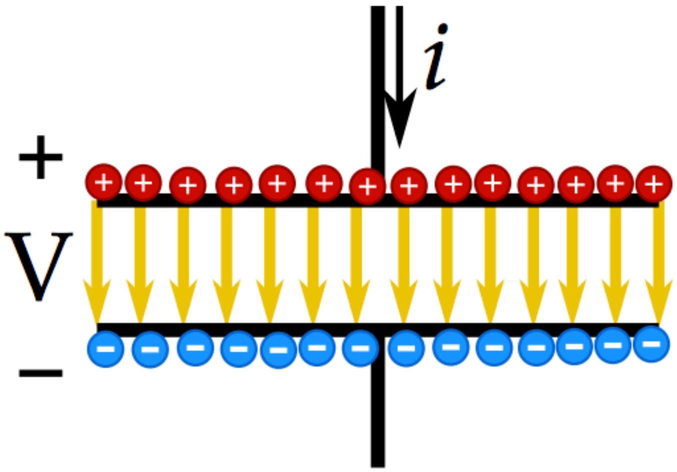
-
- Learn what a dB is, and how it is used in voltage measurement
  - Logarithmic measure of gain (V_1/V_o)
  - 10 dB is a factor of 10 in power, 20 dB is a factor of 100
- Understand what a Bode plot is, and how to use it
  - It plots of circuit's gain in dB vs. log of the frequency
- Impedance is the relationship between voltage and current
  - For a sinusoidal input
  - Z= V/i, so for a capacitor $Z = \frac{1}{2\pi FC}$

CAPACITORS

Capacitors

- Rather than relating i and V
  - Q = CV
  - Q in Coulombs
  - V in Volts
  - C in Farads

- Charge neutral
  - +Q on one lead
  - -Q on the other lead



$$Q = CV$$

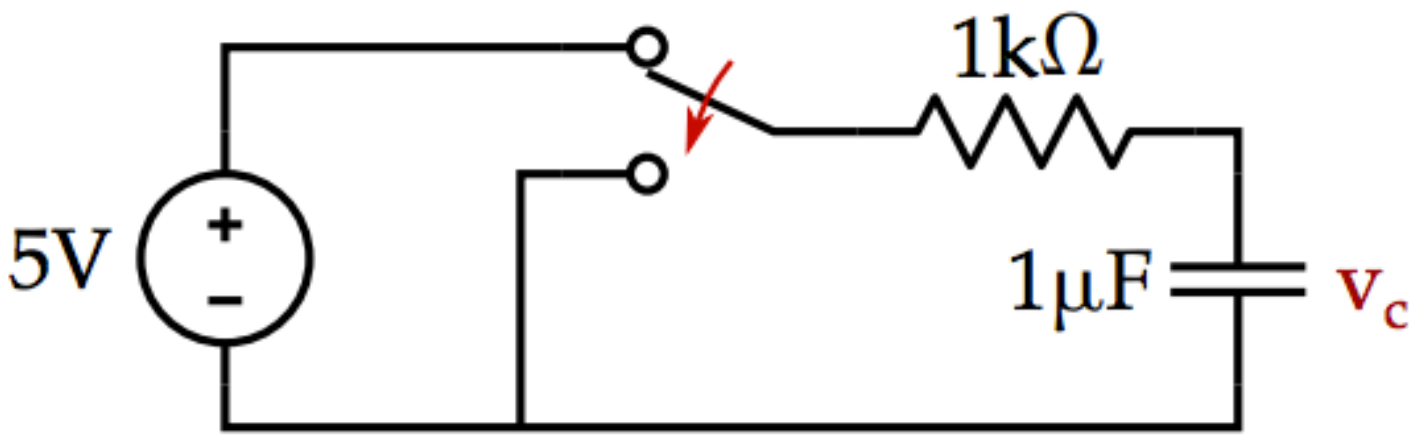
iV for a Capacitor

- How do we relate I and V though?
  - Q=CV
  - I = dQ/dt
  - dQ/dt = d(CV)/dt

- I = C dV/dt

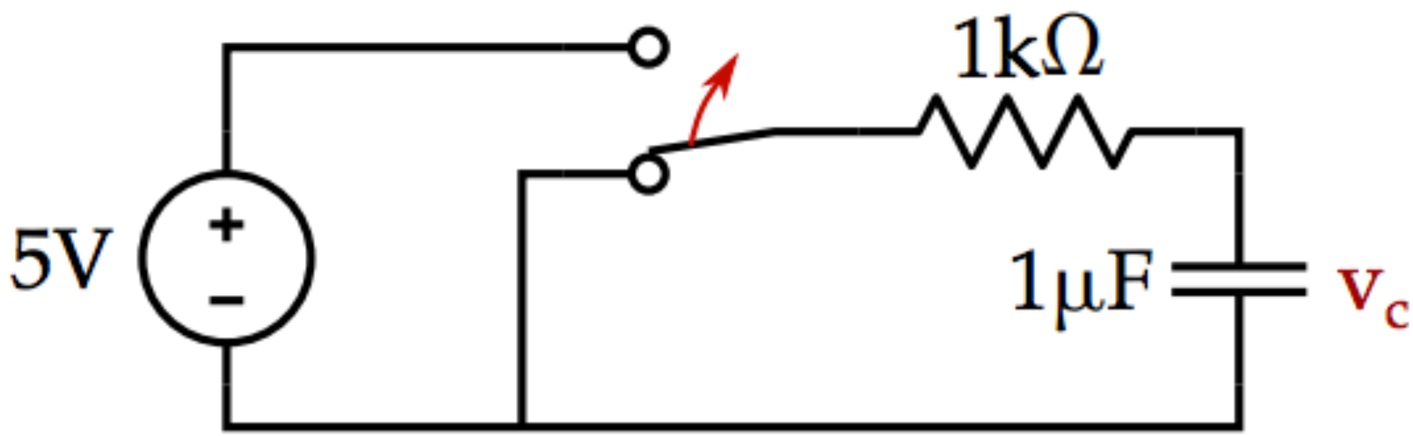
- Current = Rate of Voltage Change * Capacitance
  - If current is small, voltage changes a little bit
  - If current is large, voltage changes a lot

Capacitor Energy (Charging) – RC Circuit



- Assume Vc=0 at t=0
- At t=0, switch flips up
- Plot Vc vs t

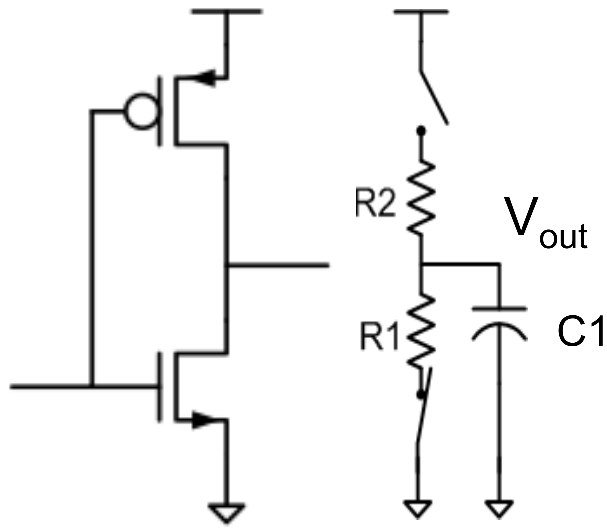
Capacitor Energy (Discharging) – RC Circuit



- Assume Vc=5V at t=0
- At t=0, switch flips down
- Plot Vc vs t

Key Ideas From The Last Lecture – RC Circuits

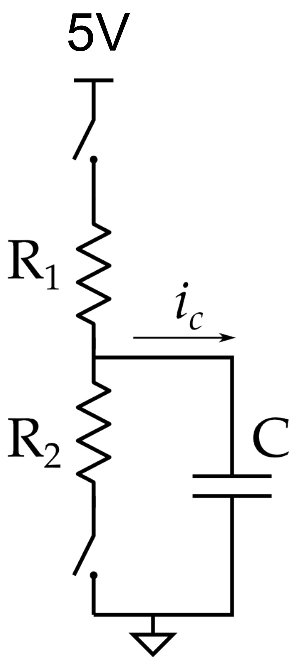
- When the input to the inverter is low, the output will be at V_{dd}
  - Right after the input rises, here is the circuit
- Want to find the capacitor voltage verses
- Just write the nodal equations:
  - We just have one node voltage, V_{OUT}
  - $i_{RES} = V_{out}/R_1$
  - $i_{CAP} = CdV_{out}/dt$



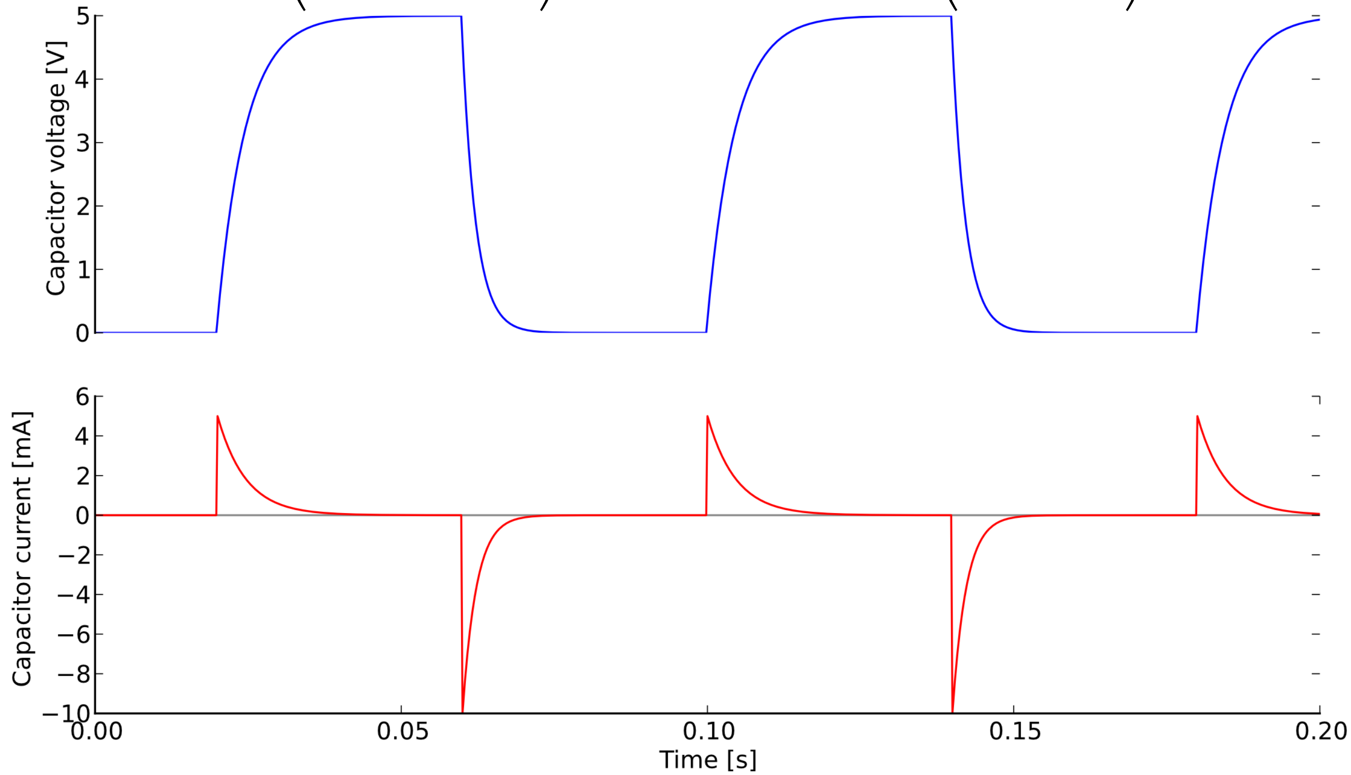
- From KCL, the sum of the currents must be zero, so

$$\frac{dV_{out}}{dt} = -\frac{V_{out}}{R_1C}$$

Key Ideas From The Last Lecture – RC Circuits



$$V_{out} = 5V\left(1 - e^{-t/R_1C}\right) \qquad V_{out} = 5V\left(e^{-t/R_1C}\right)$$



In capacitor circuits, voltages change “slowly”, while currents can be instantaneous.

PREVIOUSLY IN E40M

IMPEDANCE

Impedance

-
- Ratio of the sine wave voltage across the device
  - To the sine wave current through the device

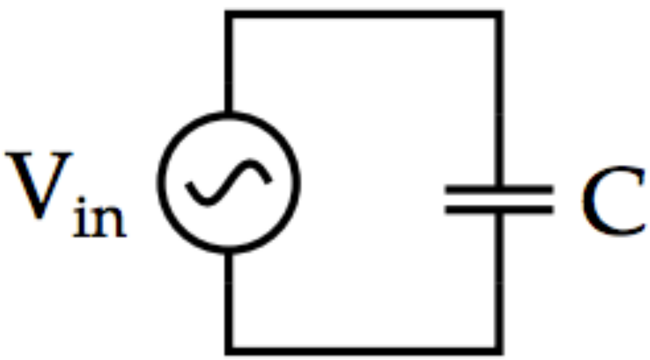
- For resistors
  - This is just the resistance, and doesn't depend on freq.

- For capacitors
  - This is a well defined quantity

$$Z = \frac{1}{2\pi F\, C}$$

    - Which depends on the frequency of the sine wave

What happens when we put sine waves into a Capacitor?

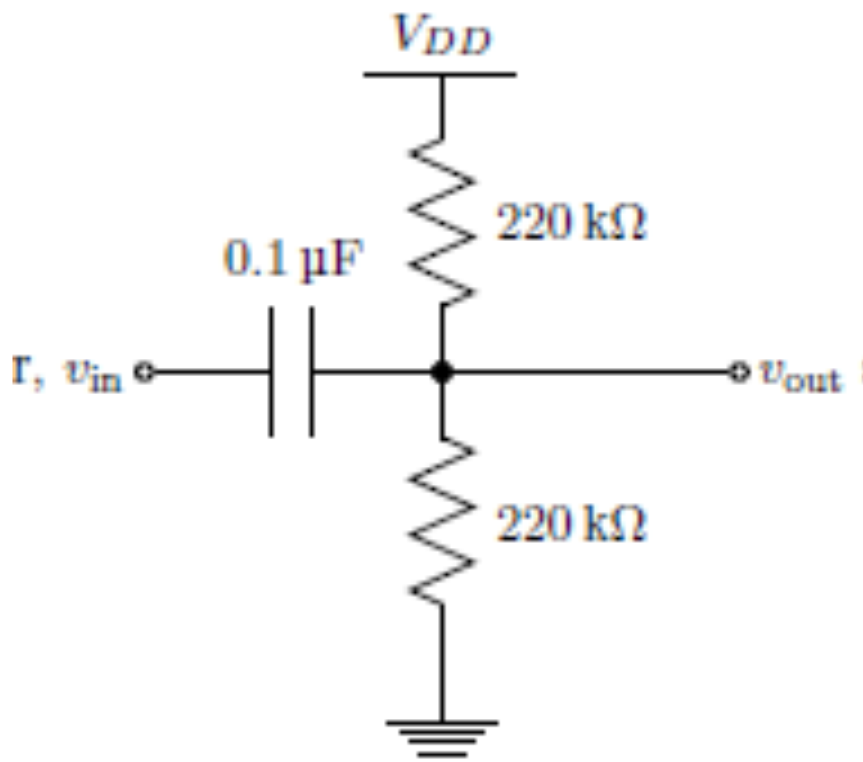


- Vin = sin(2πf t)
- Ic = C dVin/dt
- Ic = C 2πf cos(2πf t)

- |Z| = Vin/Ic = 1/(2πf C)

But How To Solve This Circuit?

- The input voltage is sound from your computer.
- Why do we need this circuit?



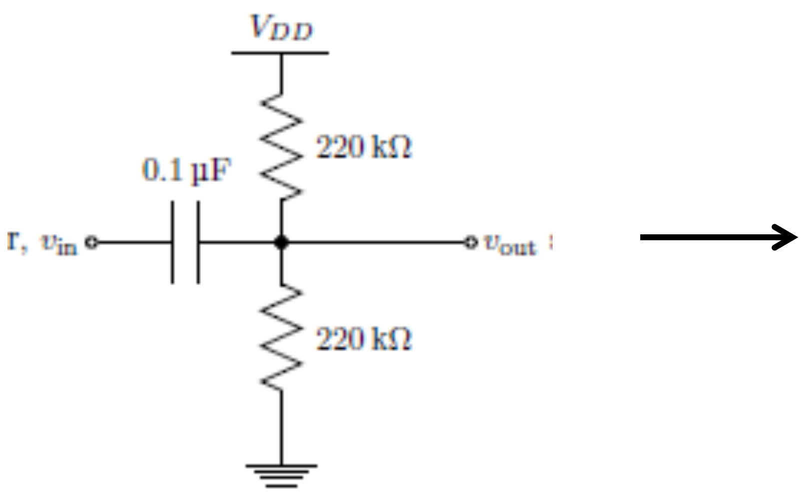
Superposition to the Rescue

- We know that sound can be represented by
  - A sum of sinewaves
- We also know that R, C are linear elements
  - So superposition holds
- Superposition says
  - The output is the sum of the response from each source
- So the output from a sound waveform
  - Is the sum of the outputs generated from each sinewave

Sinewave Driven Circuits

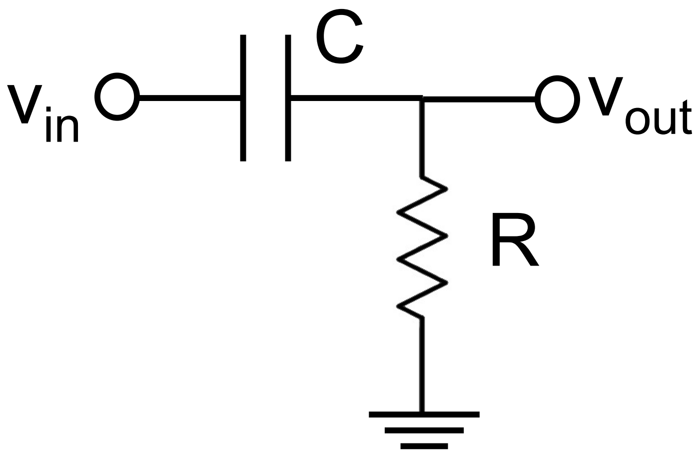
- All voltages and currents are sinusoidal
- So we really just need to figure out
  - What is the amplitude of the resulting sinewave
  - And sometimes we need the phase shift too (but not always)
- These values don't change with time
  - This problem is very similar to solving for DC voltages/currents
- In fact can solve it exactly the same way …

RC Circuit Analysis Using Impedance



- The circuit becomes just a voltage divider, and we can analyze it the same way we have analyzed resistor only circuits.
  - That's the power of using impedance!

Analyzing RC Circuits Using Impedance



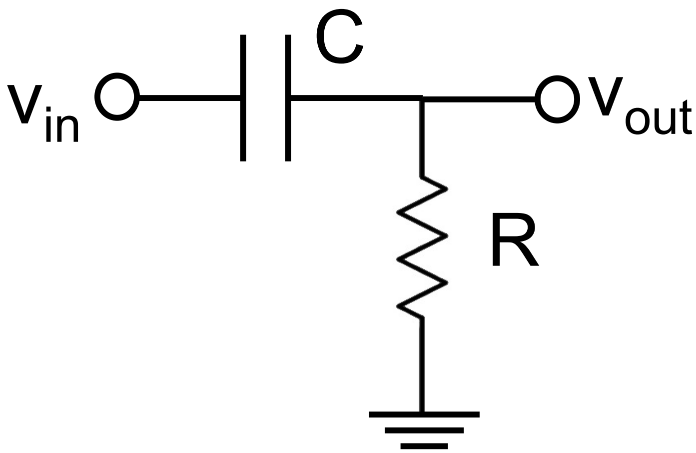
$$Z_C = \frac{1}{2\pi FC} \qquad Z_R = R$$

- If the circuit had two resistors then we would know how to analyze it

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

- What if we do the same thing but use impedances

Analyzing RC Circuits Using Impedance



$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{2\pi FC}} = \frac{2\pi FRC}{1 + 2\pi FRC}$$

- At low frequencies, (F ≈ 0), V_{out} = 0 which means that low frequencies are not passed to the output. The capacitor blocks them.
  - Recall that we used this idea earlier to calculate the DC voltage at the output.
- At high frequencies (F large), $V_{out} = V_{in}$